Complex Number

Monday, 7 November 2022 3:03 PM

M/J/2005/Q3

(i) Solve the equation $z^2 - 2iz - 5 = 0$, giving your answers in the form $x + iy$ where x and y are real. [3]

(ii) Find the modulus and argument of each root. [3]

(iii) Sketch an Argand diagram showing the points representing the roots. [1]

M/J/2009/Q7

(i) Solve the equation $z^2 + (2\sqrt{3})iz - 4 = 0$, giving your answers in the form $x + iy$, where x and y are real. [3]

(ii) Sketch an Argand diagram showing the points representing the roots. [1]

(iii) Find the modulus and argument of each root. [3]

(iv) Show that the origin and the points representing the roots are the vertices of an equilateral triangle. [1]

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The complex number z is given by

$$z = (\sqrt{3}) + \mathrm{i}.$$

(i) Find the modulus and argument of z. [2]

(ii) The complex conjugate of z is denoted by z^*. Showing your working, express in the form $x + \mathrm{i}y$, where x and y are real,

(a) $2z + z^*$,

(b) $\dfrac{\mathrm{i}z^*}{z}$.

[4]

(iii) On a sketch of an Argand diagram with origin O, show the points A and B representing the complex numbers z and $\mathrm{i}z^*$ respectively. Prove that angle $AOB = \frac{1}{6}\pi$. [3]

M/J/2018/Q7

Throughout this question the use of a calculator is not permitted.

The complex numbers $-3\sqrt{3} + \mathrm{i}$ and $\sqrt{3} + 2\mathrm{i}$ are denoted by u and v respectively.

(i) Find, in the form $x + \mathrm{i}y$, where x and y are real and exact, the complex numbers uv and $\frac{u}{v}$. [5]

(ii) On a sketch of an Argand diagram with origin O, show the points A and B representing the complex numbers u and v respectively. Prove that angle $AOB = \frac{2}{3}\pi$. [3]

The complex number u is defined by

$$u = \frac{1 + 2i}{1 - 3i}.$$

(i) Express u in the form $x + iy$, where x and y are real. [3]

(ii) Show on a sketch of an Argand diagram the points A, B and C representing the complex numbers u, $1 + 2i$ and $1 - 3i$ respectively. [2]

(iii) By considering the arguments of $1 + 2i$ and $1 - 3i$, show that

$$\tan^{-1} 2 + \tan^{-1} 3 = \tfrac{3}{4}\pi.$$ [3]

M/J/2006/Q7

The complex number $2 + i$ is denoted by u. Its complex conjugate is denoted by u^*.

(i) Show, on a sketch of an Argand diagram with origin O, the points A, B and C representing the complex numbers u, u^* and $u + u^*$ respectively. Describe in geometrical terms the relationship between the four points O, A, B and C. [4]

(ii) Express $\frac{u}{u^*}$ in the form $x + iy$, where x and y are real. [3]

(iii) By considering the argument of $\frac{u}{u^*}$, or otherwise, prove that

$$\tan^{-1}\left(\tfrac{4}{3}\right) = 2\tan^{-1}\left(\tfrac{1}{2}\right).$$ [2]

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O/N/2005/Q7

The equation $2x^3 + x^2 + 25 = 0$ has one real root and two complex roots.

(i) Verify that $1 + 2i$ is one of the complex roots. [3]

(ii) Write down the other complex root of the equation. [1]

(iii) Sketch an Argand diagram showing the point representing the complex number $1 + 2i$. Show on the same diagram the set of points representing the complex numbers z which satisfy

$$|z| = |z - 1 - 2i|.$$ [4]

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The complex number u is given by

$$u = \frac{3 + \mathrm{i}}{2 - \mathrm{i}}.$$

(i) Express u in the form $x + \mathrm{i}y$, where x and y are real. [3]

(ii) Find the modulus and argument of u. [2]

(iii) Sketch an Argand diagram showing the point representing the complex number u. Show on the same diagram the locus of the point representing the complex number z such that $|z - u| = 1$. [3]

The complex number $\frac{2}{-1+\mathrm{i}}$ is denoted by u.

(i) Find the modulus and argument of u and u^2. [6]

(ii) Sketch an Argand diagram showing the points representing the complex numbers u and u^2. Shade the region whose points represent the complex numbers z which satisfy both the inequalities $|z| < 2$ and $|z - u^2| < |z - u|$. [4]

(a) The complex number u is defined by $u = \frac{5}{a + 2\mathrm{i}}$, where the constant a is real.

(i) Express u in the form $x + \mathrm{i}y$, where x and y are real. [2]

(ii) Find the value of a for which $\arg(u^*) = \frac{3}{4}\pi$, where u^* denotes the complex conjugate of u. [3]

(b) On a sketch of an Argand diagram, shade the region whose points represent complex numbers z which satisfy both the inequalities $|z| < 2$ and $|z| < |z - 2 - 2\mathrm{i}|$. [4]

O/N/2009/Q7

The complex numbers $-2 + \mathrm{i}$ and $3 + \mathrm{i}$ are denoted by u and v respectively.

(i) Find, in the form $x + \mathrm{i}y$, the complex numbers

(a) $u + v$, [1]

(b) $\frac{u}{v}$, showing all your working. [3]

(ii) State the argument of $\frac{u}{v}$. [1]

In an Argand diagram with origin O, the points A, B and C represent the complex numbers u, v and $u + v$ respectively.

(iii) Prove that angle $AOB = \frac{3}{4}\pi$. [2]

(iv) State fully the geometrical relationship between the line segments OA and BC. [2]

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The complex number $3 - \mathrm{i}$ is denoted by u. Its complex conjugate is denoted by u^*.

(i) On an Argand diagram with origin O, show the points A, B and C representing the complex numbers u, u^* and $u^* - u$ respectively. What type of quadrilateral is $OABC$? [4]

(ii) Showing your working and without using a calculator, express $\dfrac{u^*}{u}$ in the form $x + \mathrm{i}y$, where x and y are real. [3]

(iii) By considering the argument of $\dfrac{u^*}{u}$, prove that

$$\tan^{-1}\left(\tfrac{3}{4}\right) = 2\tan^{-1}\left(\tfrac{1}{3}\right).$$ [3]

(a) Find the complex number z satisfying the equation

$$z + \frac{iz}{z^*} - 2 = 0,$$

where z^* denotes the complex conjugate of z. Give your answer in the form $x + iy$, where x and y are real. [5]

(b) **(i)** On a single Argand diagram sketch the loci given by the equations $|z - 2i| = 2$ and $\text{Im}\, z = 3$, where $\text{Im}\, z$ denotes the imaginary part of z. [2]

(ii) In the first quadrant the two loci intersect at the point P. Find the exact argument of the complex number represented by P. [2]

O/N/2007/Q8

(a) The complex number z is given by $z = \dfrac{4-3\text{i}}{1-2\text{i}}$.

(i) Express z in the form $x + \text{i}y$, where x and y are real. [2]

(ii) Find the modulus and argument of z. [2]

(b) Find the two square roots of the complex number $5 - 12\text{i}$, giving your answers in the form $x + \text{i}y$, where x and y are real. [6]

O/N/2011/Q10

(a) Showing your working, find the two square roots of the complex number $1 - (2\sqrt{6})\text{i}$. Give your answers in the form $x + \text{i}y$, where x and y are exact. [5]

(b) On a sketch of an Argand diagram, shade the region whose points represent the complex numbers z which satisfy the inequality $|z - 3\text{i}| \leqslant 2$. Find the greatest value of $\arg z$ for points in this region. [5]

M/J/2013/Q9

(a) The complex number w is such that $\operatorname{Re} w > 0$ and $w + 3w^* = \mathrm{i}w^2$, where w^* denotes the complex conjugate of w. Find w, giving your answer in the form $x + \mathrm{i}y$, where x and y are real. [5]

(b) On a sketch of an Argand diagram, shade the region whose points represent complex numbers z which satisfy both the inequalities $|z - 2\mathrm{i}| \leqslant 2$ and $0 \leqslant \arg(z + 2) \leqslant \frac{1}{4}\pi$. Calculate the greatest value of $|z|$ for points in this region, giving your answer correct to 2 decimal places. [6]

O/N/2013/Q8

Throughout this question the use of a calculator is not permitted.

(a) The complex numbers u and v satisfy the equations

$$u + 2v = 2\text{i} \quad \text{and} \quad \text{i}u + v = 3.$$

Solve the equations for u and v, giving both answers in the form $x + \text{i}y$, where x and y are real. [5]

(b) On an Argand diagram, sketch the locus representing complex numbers z satisfying $|z + \text{i}| = 1$ and the locus representing complex numbers w satisfying $\arg(w - 2) = \frac{3}{4}\pi$. Find the least value of $|z - w|$ for points on these loci. [5]

M/J/2015/Q7

The complex number u is given by $u = -1 + (4\sqrt{3})\text{i}$.

(i) Without using a calculator and showing all your working, find the two square roots of u. Give your answers in the form $a + \text{i}b$, where the real numbers a and b are exact. [5]

(ii) On an Argand diagram, sketch the locus of points representing complex numbers z satisfying the relation $|z - u| = 1$. Determine the greatest value of $\arg z$ for points on this locus. [4]

M/J/2019/Q5

Throughout this question the use of a calculator is not permitted.

It is given that the complex number $-1 + (\sqrt{3})\mathrm{i}$ is a root of the equation

$$kx^3 + 5x^2 + 10x + 4 = 0,$$

where k is a real constant.

(i) Write down another root of the equation. [1]

(ii) Find the value of k and the third root of the equation. [6]

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The complex number $1 + (\sqrt{2})\mathrm{i}$ is denoted by u. The polynomial $x^4 + x^2 + 2x + 6$ is denoted by $\mathrm{p}(x)$.

(i) Showing your working, verify that u is a root of the equation $\mathrm{p}(x) = 0$, and write down a second complex root of the equation. [4]

(ii) Find the other two roots of the equation $\mathrm{p}(x) = 0$. [6]

M/J/2017/Q6

Throughout this question the use of a calculator is not permitted.

The complex number $2 - \mathrm{i}$ is denoted by u.

(i) It is given that u is a root of the equation $x^3 + ax^2 - 3x + b = 0$, where the constants a and b are real. Find the values of a and b. [4]

(ii) On a sketch of an Argand diagram, shade the region whose points represent complex numbers z satisfying both the inequalities $|z - u| < 1$ and $|z| < |z + \mathrm{i}|$. [4]

M/J/2014/Q7

(a) It is given that $-1 + (\sqrt{5})\text{i}$ is a root of the equation $z^3 + 2z + a = 0$, where a is real. Showing your working, find the value of a, and write down the other complex root of this equation. [4]

(b) The complex number w has modulus 1 and argument 2θ radians. Show that $\frac{w-1}{w+1} = \text{i} \tan \theta$. [4]

The variable complex number z is given by

$$z = 1 + \cos 2\theta + \mathrm{i}\sin 2\theta,$$

where θ takes all values in the interval $-\frac{1}{2}\pi < \theta < \frac{1}{2}\pi$.

(i) Show that the modulus of z is $2\cos\theta$ and the argument of z is θ. [6]

(ii) Prove that the real part of $\frac{1}{z}$ is constant. [3]

M/J/2008/Q5

The variable complex number z is given by

$$z = 2\cos\theta + i(1 - 2\sin\theta),$$

where θ takes all values in the interval $-\pi < \theta \leqslant \pi$.

(i) Show that $|z - i| = 2$, for all values of θ. Hence sketch, in an Argand diagram, the locus of the point representing z. [3]

(ii) Prove that the real part of $\dfrac{1}{z + 2 - i}$ is constant for $-\pi < \theta < \pi$. [4]

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O/N/2018/Q9

(a) **(i)** Without using a calculator, express the complex number $\frac{2 + 6i}{1 - 2i}$ in the form $x + iy$, where x and y are real. [2]

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(ii) Hence, without using a calculator, express $\dfrac{2+6\mathrm{i}}{1-2\mathrm{i}}$ in the form $r(\cos\theta + \mathrm{i}\sin\theta)$, where $r > 0$ and $-\pi < \theta \leqslant \pi$, giving the exact values of r and θ. [3]

(b) On a sketch of an Argand diagram, shade the region whose points represent complex numbers z satisfying both the inequalities $|z - 3\mathrm{i}| \leqslant 1$ and $\operatorname{Re} z \leqslant 0$, where $\operatorname{Re} z$ denotes the real part of z. Find the greatest value of $\arg z$ for points in this region, giving your answer in radians correct to 2 decimal places. [5]