Trigonometry

O/N/2010/Q3

Solve the equation

$$\cos(\theta + 60^\circ) = 2\sin\theta,$$

giving all solutions in the interval $0^\circ \leqslant \theta \leqslant 360^\circ$. [5]

O/N/2012/Q3

Solve the equation

$$\sin(\theta + 45^\circ) = 2\cos(\theta - 30^\circ),$$

giving all solutions in the interval $0^\circ < \theta < 180^\circ$. [5]

O/N/2018/Q2

Showing all necessary working, solve the equation $\sin(\theta - 30^\circ) + \cos\theta = 2\sin\theta$, for $0^\circ < \theta < 180^\circ$. [4]

M/J/2014/Q3

Solve the equation

$$\cos(x + 30°) = 2\cos x,$$

giving all solutions in the interval $-180° < x < 180°$. [5]

O/N/2007Q5

(i) Show that the equation

$$\tan(45^\circ + x) - \tan x = 2$$

can be written in the form

$$\tan^2 x + 2\tan x - 1 = 0.$$ [3]

(ii) Hence solve the equation

$$\tan(45^\circ + x) - \tan x = 2,$$

giving all solutions in the interval $0^\circ \leqslant x \leqslant 180^\circ$. [4]

O/N/2017/Q3 ANS;16.8

By expressing the equation $\tan(\theta + 60°) + \tan(\theta - 60°) = \cot\theta$ in terms of $\tan\theta$ only, solve the equation for $0° < \theta < 90°$. [5]

(i) Show that the equation $\tan(30^\circ + \theta) = 2\tan(60^\circ - \theta)$ can be written in the form

$$\tan^2\theta + (6\sqrt{3})\tan\theta - 5 = 0.$$ [4]

(ii) Hence, or otherwise, solve the equation

$$\tan(30^\circ + \theta) = 2\tan(60^\circ - \theta),$$

for $0^\circ \leqslant \theta \leqslant 180^\circ$. [3]

Showing all necessary working, solve the equation $\cot\theta + \cot(\theta + 45^\circ) = 2$, for $0^\circ < \theta < 180^\circ$. [5]

The angles θ and ϕ lie between $0°$ and $180°$, and are such that

$$\tan(\theta - \phi) = 3 \quad \text{and} \quad \tan\theta + \tan\phi = 1.$$

Find the possible values of θ and ϕ. [6]

(i) Express $7\cos\theta + 24\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$, giving the exact value of R and the value of α correct to 2 decimal places. [3]

(ii) Hence solve the equation

$$7\cos\theta + 24\sin\theta = 15,$$

giving all solutions in the interval $0° \leqslant \theta \leqslant 360°$. [4]

O/N/2005/Q5

By expressing $8 \sin \theta - 6 \cos \theta$ in the form $R \sin(\theta - \alpha)$, solve the equation

$$8 \sin \theta - 6 \cos \theta = 7,$$

for $0° \leqslant \theta \leqslant 360°$. [7]

(i) Express $3\sin\theta + 2\cos\theta$ in the form $R\sin(\theta + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$, stating the exact value of R and giving the value of α correct to 2 decimal places. [3]

(ii) Hence solve the equation

$$3\sin\theta + 2\cos\theta = 1,$$

for $0° < \theta < 180°$. [3]

O/N/2019/Q4

(i) Express $(\sqrt{6})\sin x + \cos x$ in the form $R\sin(x + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. State the exact value of R and give α correct to 3 decimal places. [3]

(ii) Hence solve the equation $(\sqrt{6})\sin 2\theta + \cos 2\theta = 2$, for $0° < \theta < 180°$. [4]

O/N/2011/Q6

(i) Express $\cos x + 3\sin x$ in the form $R\cos(x - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$, giving the exact value of R and the value of α correct to 2 decimal places. [3]

(ii) Hence solve the equation $\cos 2\theta + 3\sin 2\theta = 2$, for $0° < \theta < 90°$. [5]

(i) Express $5\sin x + 12\cos x$ in the form $R\sin(x + \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$, giving the value of α correct to 2 decimal places. [3]

(ii) Hence solve the equation

$$5\sin 2\theta + 12\cos 2\theta = 11,$$

giving all solutions in the interval $0^\circ < \theta < 180^\circ$. [5]

(i) By first expanding $\cos(x + 45°)$, express $\cos(x + 45°) - (\sqrt{2})\sin x$ in the form $R\cos(x + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. Give the value of R correct to 4 significant figures and the value of α correct to 2 decimal places. [5]

(ii) Hence solve the equation

$$\cos(x + 45°) - (\sqrt{2})\sin x = 2,$$

for $0° < x < 360°$. [4]

Showing all necessary working, solve the equation $\cot 2\theta = 2\tan\theta$ for $0° < \theta < 180°$. [5]

M/J/2012/Q4

Solve the equation

$$\operatorname{cosec} 2\theta = \sec\theta + \cot\theta,$$

giving all solutions in the interval $0^\circ < \theta < 360^\circ$. [6]

M/J/2011/Q3

Solve the equation

$$\cos\theta + 4\cos 2\theta = 3,$$

giving all solutions in the interval $0^\circ \leqslant \theta \leqslant 180^\circ$. [5]

O/N/2006/Q2

Solve the equation

$$\tan x \tan 2x = 1,$$

giving all solutions in the interval $0° < x < 180°$. [4]

M/J/2017/Q3

(i) Express the equation $\cot\theta - 2\tan\theta = \sin 2\theta$ in the form $a\cos^4\theta + b\cos^2\theta + c = 0$, where a, b and c are constants to be determined. [3]

(ii) Hence solve the equation $\cot\theta - 2\tan\theta = \sin 2\theta$ for $90° < \theta < 180°$. [2]

(i) Prove the identity $\operatorname{cosec} 2\theta + \cot 2\theta \equiv \cot \theta$. [3]

(ii) Hence solve the equation $\operatorname{cosec} 2\theta + \cot 2\theta = 2$, for $0° \leqslant \theta \leqslant 360°$. [2]

(i) Prove the identity

$$\cos 4\theta + 4\cos 2\theta \equiv 8\cos^4\theta - 3.$$ [4]

(ii) Hence solve the equation

$$\cos 4\theta + 4\cos 2\theta = 2,$$

for $0° \leqslant \theta \leqslant 360°$. [4]

(i) By first expanding $\sin(2\theta + \theta)$, show that

$$\sin 3\theta = 3\sin\theta - 4\sin^3\theta.$$ [4]

(ii) Show that, after making the substitution $x = \dfrac{2\sin\theta}{\sqrt{3}}$, the equation $x^3 - x + \frac{1}{6}\sqrt{3} = 0$ can be written in the form $\sin 3\theta = \frac{3}{4}$. [1]

(iii) Hence solve the equation

$$x^3 - x + \tfrac{1}{6}\sqrt{3} = 0,$$

giving your answers correct to 3 significant figures. [4]