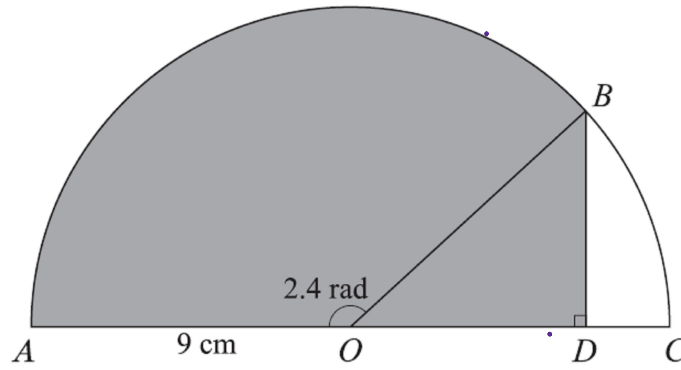


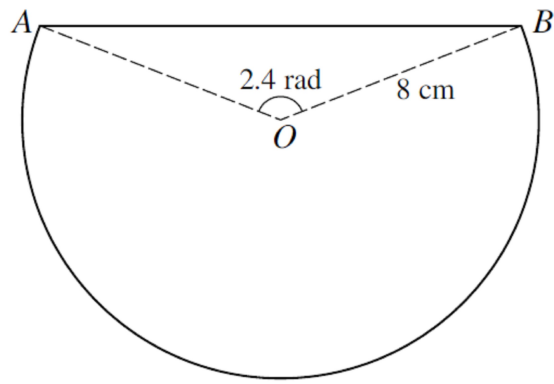
CIRCULAR MEASURE

M/J/2005/Q8



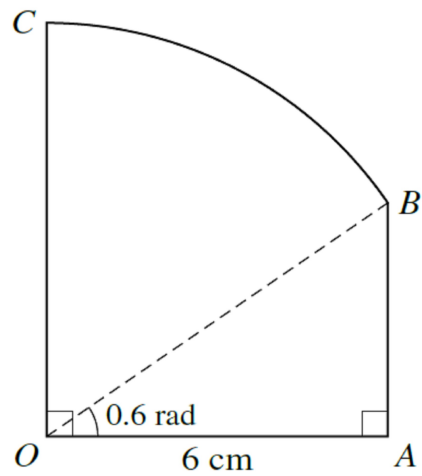
In the diagram, ABC is a semicircle, centre O and radius 9 cm. The line BD is perpendicular to the diameter AC and angle $AOB = 2.4$ radians.

- (i) Show that $BD = 6.08$ cm, correct to 3 significant figures. [2]
- (ii) Find the perimeter of the shaded region. [3]
- (iii) Find the area of the shaded region. [3]



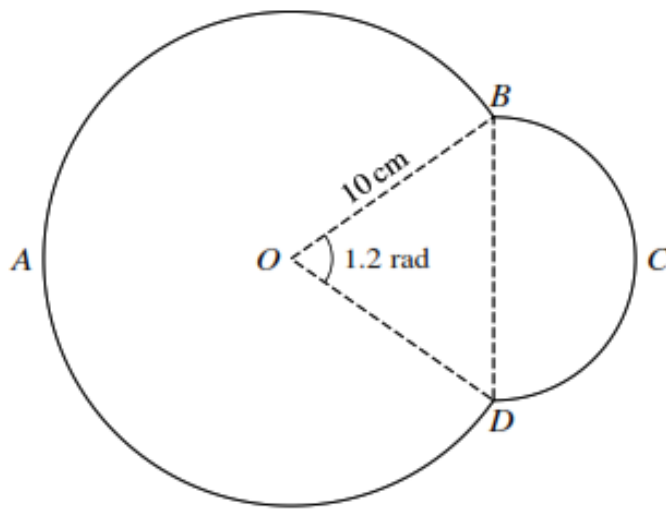
The diagram shows a metal plate made by removing a segment from a circle with centre O and radius 8 cm . The line AB is a chord of the circle and angle $AOB = 2.4$ radians. Find

- (i) the length of AB , [2]
- (ii) the perimeter of the plate, [3]
- (iii) the area of the plate. [3]



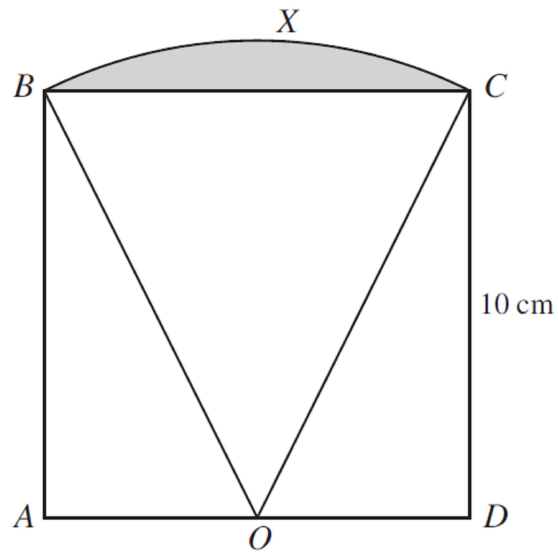
The diagram shows a metal plate $OABC$, consisting of a right-angled triangle OAB and a sector OBC of a circle with centre O . Angle $AOB = 0.6$ radians, $OA = 6$ cm and OA is perpendicular to OC .

- (i) Show that the length of OB is 7.270 cm, correct to 3 decimal places. [1]
- (ii) Find the perimeter of the metal plate. [3]
- (iii) Find the area of the metal plate. [3]



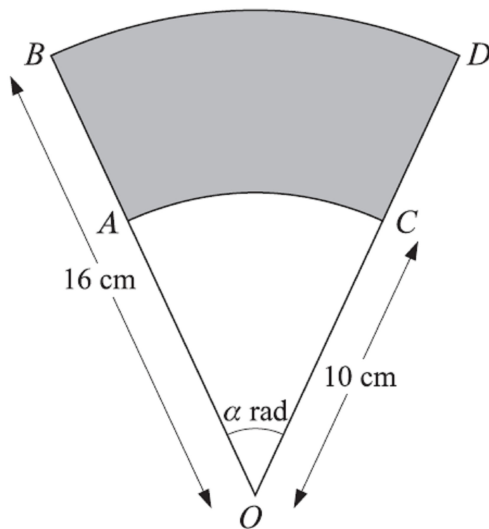
The diagram shows a metal plate $ABCD$ made from two parts. The part BCD is a semicircle. The part DAB is a segment of a circle with centre O and radius 10 cm. Angle BOD is 1.2 radians.

- (i) Show that the radius of the semicircle is 5.646 cm, correct to 3 decimal places. [2]
- (ii) Find the perimeter of the metal plate. [3]
- (iii) Find the area of the metal plate. [3]



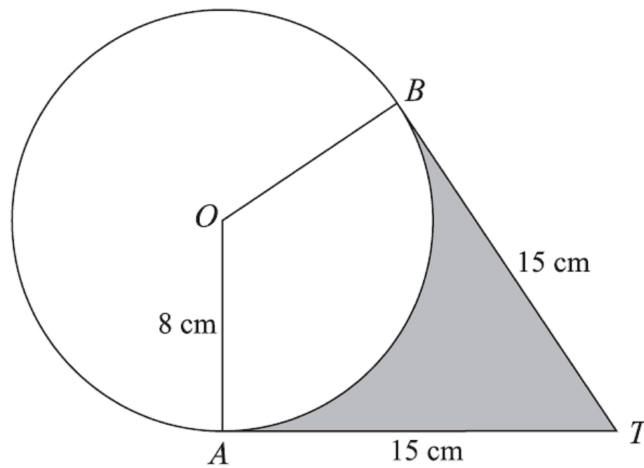
The diagram shows a square $ABCD$ of side 10 cm. The mid-point of AD is O and BXC is an arc of a circle with centre O .

- (i) Show that angle BOC is 0.9273 radians, correct to 4 decimal places. [2]
- (ii) Find the perimeter of the shaded region. [3]
- (iii) Find the area of the shaded region. [2]



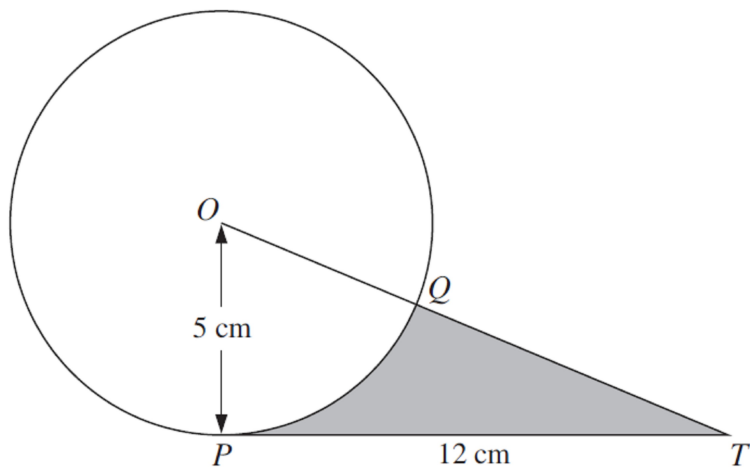
In the diagram, OAB and OCD are radii of a circle, centre O and radius 16 cm. Angle $AOC = \alpha$ radians. AC and BD are arcs of circles, centre O and radii 10 cm and 16 cm respectively.

- (i) In the case where $\alpha = 0.8$, find the area of the shaded region. [2]
- (ii) Find the value of α for which the perimeter of the shaded region is 28.9 cm. [3]



The diagram shows a circle with centre O and radius 8 cm . Points A and B lie on the circle. The tangents at A and B meet at the point T , and $AT = BT = 15\text{ cm}$.

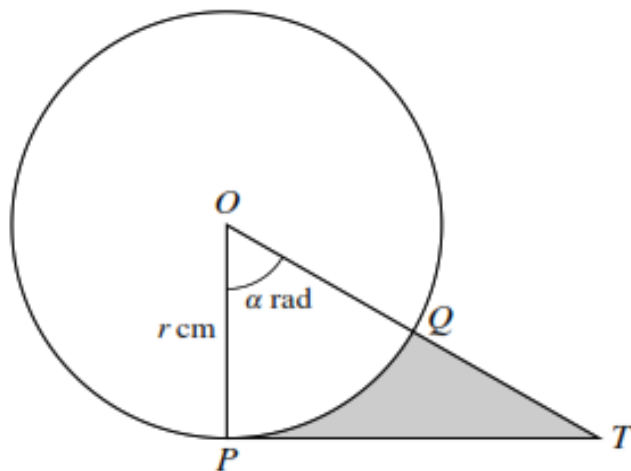
- (i) Show that angle AOB is 2.16 radians, correct to 3 significant figures. [3]
- (ii) Find the perimeter of the shaded region. [2]
- (iii) Find the area of the shaded region. [3]



The diagram shows a circle with centre O and radius 5 cm. The point P lies on the circle, PT is a tangent to the circle and $PT = 12$ cm. The line OT cuts the circle at the point Q .

(i) Find the perimeter of the shaded region. [4]

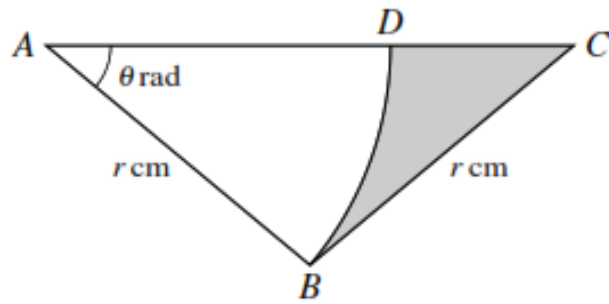
(ii) Find the area of the shaded region. [3]



The diagram shows a circle with radius r cm and centre O . The line PT is the tangent to the circle at P and angle $POT = \alpha$ radians. The line OT meets the circle at Q .

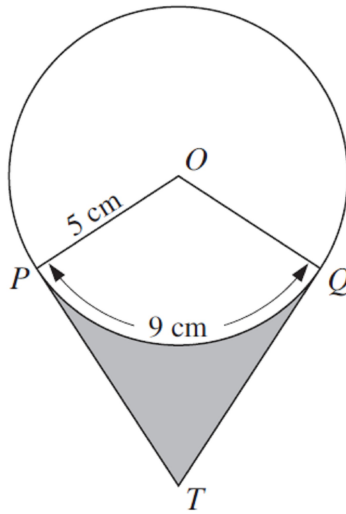
(i) Express the perimeter of the shaded region PQT in terms of r and α . [3]

(ii) In the case where $\alpha = \frac{1}{3}\pi$ and $r = 10$, find the area of the shaded region correct to 2 significant figures. [3]



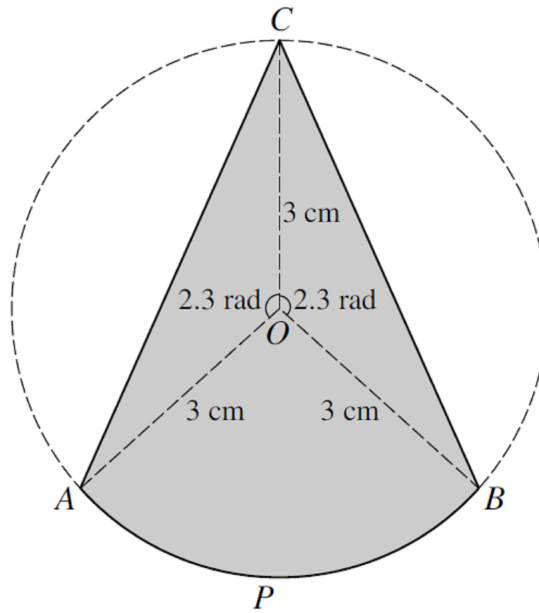
In the diagram, ABC is an isosceles triangle with $AB = BC = r$ cm and angle $BAC = \theta$ radians. The point D lies on AC and ABD is a sector of a circle with centre A .

- (a) Express the area of the shaded region in terms of r and θ . [3]
- (b) In the case where $r = 10$ and $\theta = 0.6$, find the perimeter of the shaded region. [4]



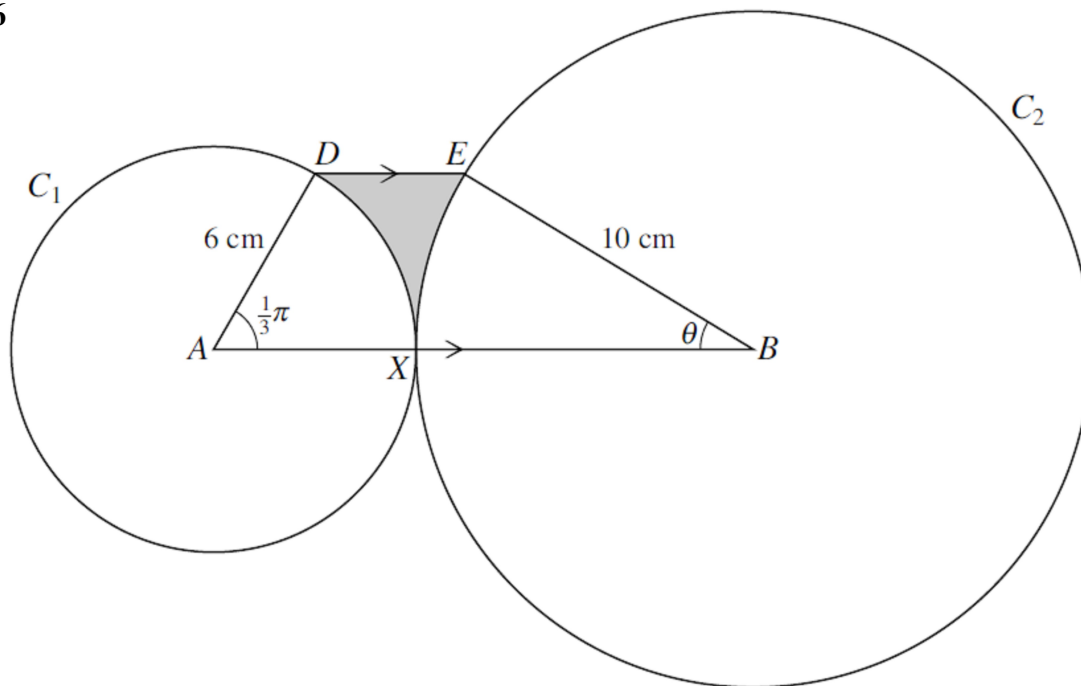
In the diagram, the circle has centre O and radius 5 cm . The points P and Q lie on the circle, and the arc length PQ is 9 cm . The tangents to the circle at P and Q meet at the point T . Calculate

- (i) angle POQ in radians, [2]
- (ii) the length of PT , [3]
- (iii) the area of the shaded region. [3]



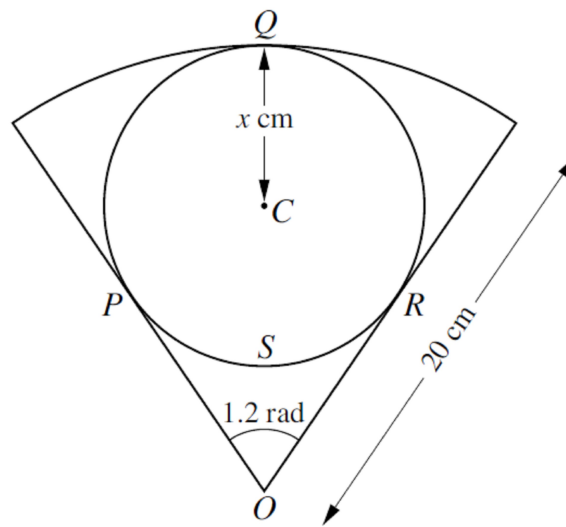
The diagram shows points A , C , B , P on the circumference of a circle with centre O and radius 3 cm. Angle $AOC = \text{angle } BOC = 2.3$ radians.

- (i) Find angle AOB in radians, correct to 4 significant figures. [1]
- (ii) Find the area of the shaded region $ACBP$, correct to 3 significant figures. [4]



The diagram shows a circle C_1 touching a circle C_2 at a point X . Circle C_1 has centre A and radius 6 cm, and circle C_2 has centre B and radius 10 cm. Points D and E lie on C_1 and C_2 respectively and DE is parallel to AB . Angle $DAX = \frac{1}{3}\pi$ radians and angle $EBX = \theta$ radians.

- (i) By considering the perpendicular distances of D and E from AB , show that the exact value of θ is $\sin^{-1}\left(\frac{3\sqrt{3}}{10}\right)$. [3]
- (ii) Find the perimeter of the shaded region, correct to 4 significant figures. [5]



The diagram shows a sector of a circle with centre O and radius 20 cm. A circle with centre C and radius x cm lies within the sector and touches it at P , Q and R . Angle $POR = 1.2$ radians.

- (i) Show that $x = 7.218$, correct to 3 decimal places. [4]
- (ii) Find the total area of the three parts of the sector lying outside the circle with centre C . [2]
- (iii) Find the perimeter of the region $OPSR$ bounded by the arc PSR and the lines OP and OR . [4]

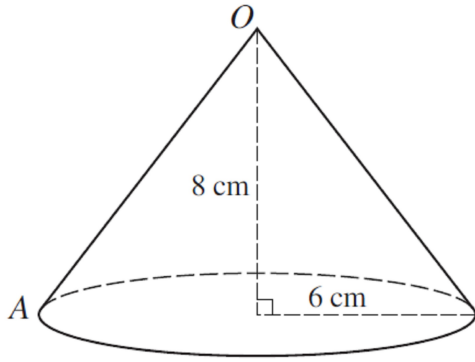


Fig. 1

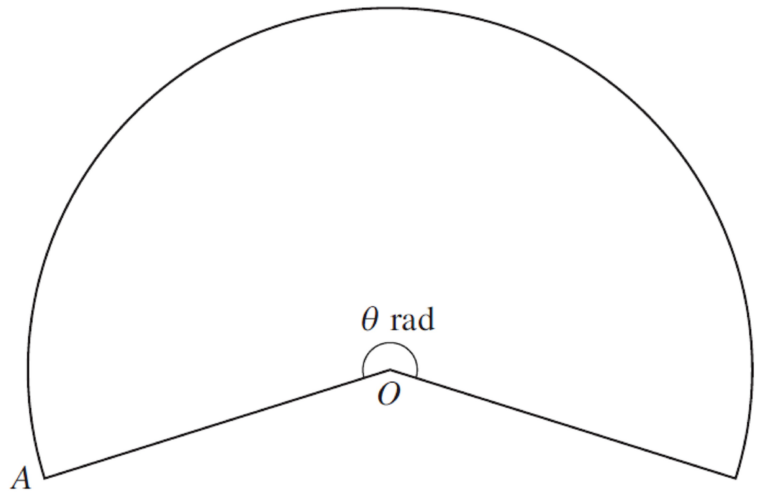
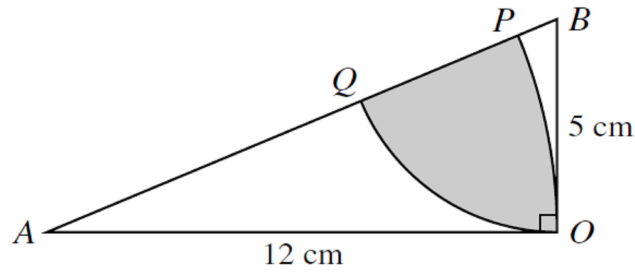


Fig. 2

Fig. 1 shows a hollow cone with no base, made of paper. The radius of the cone is 6 cm and the height is 8 cm. The paper is cut from A to O and opened out to form the sector shown in Fig. 2. The circular bottom edge of the cone in Fig. 1 becomes the arc of the sector in Fig. 2. The angle of the sector is θ radians. Calculate

- (i) the value of θ , [4]
- (ii) the area of paper needed to make the cone. [2]

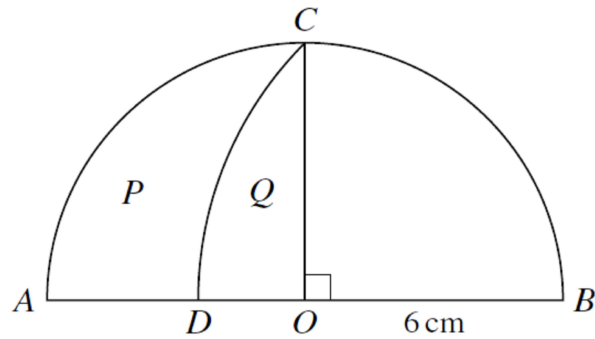
O/N/2014/Q2



The diagram shows a triangle AOB in which OA is 12 cm, OB is 5 cm and angle AOB is a right angle. Point P lies on AB and OP is an arc of a circle with centre A . Point Q lies on AB and OQ is an arc of a circle with centre B .

(i) Show that angle BAO is 0.3948 radians, correct to 4 decimal places. [1]

(ii) Calculate the area of the shaded region. [5]



The diagram shows a semicircle with centre O and radius 6 cm. The radius OC is perpendicular to the diameter AB . The point D lies on AB , and DC is an arc of a circle with centre B .

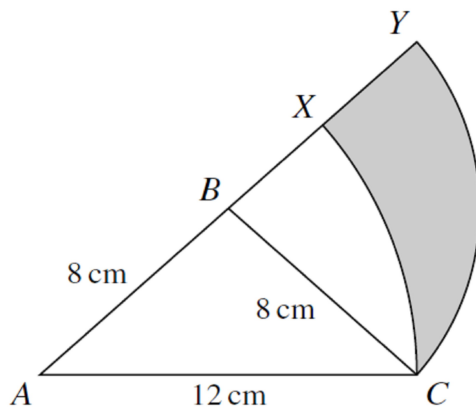
(i) Calculate the length of the arc DC . [3]

(ii) Find the value of

$$\frac{\text{area of region } P}{\text{area of region } Q},$$

giving your answer correct to 3 significant figures.

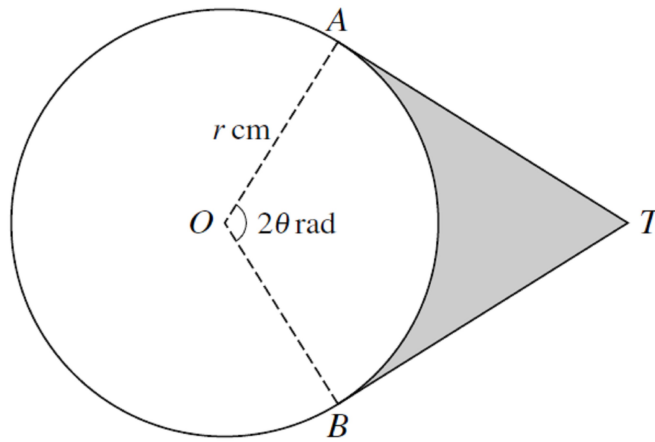
[4]



The diagram shows an isosceles triangle ACB in which $AB = BC = 8$ cm and $AC = 12$ cm. The arc XC is part of a circle with centre A and radius 12 cm, and the arc YC is part of a circle with centre B and radius 8 cm. The points A , B , X and Y lie on a straight line.

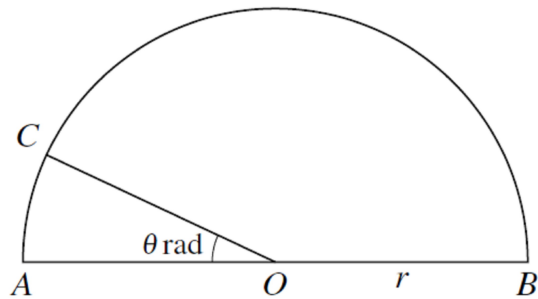
(i) Show that angle $CBY = 1.445$ radians, correct to 4 significant figures. [3]

(ii) Find the perimeter of the shaded region. [4]

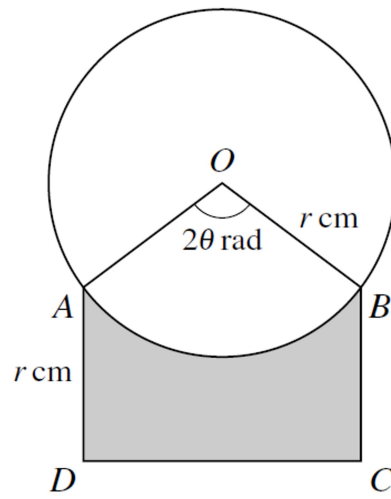


The diagram shows a circle with centre O and radius r cm. Points A and B lie on the circle and angle $AOB = 2\theta$ radians. The tangents to the circle at A and B meet at T .

- (i) Express the perimeter of the shaded region in terms of r and θ . [3]
- (ii) In the case where $r = 5$ and $\theta = 1.2$, find the area of the shaded region. [4]

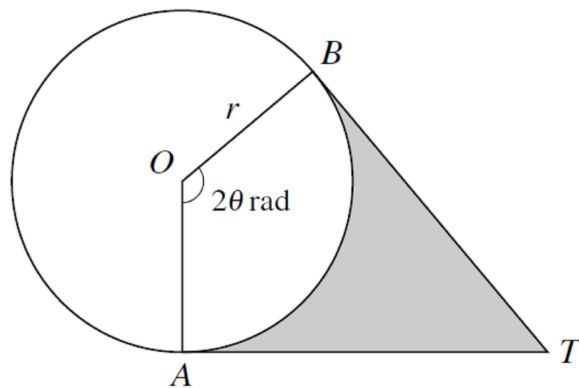


The diagram shows a semicircle with diameter AB , centre O and radius r . The point C lies on the circumference and angle $AOC = \theta$ radians. The perimeter of sector BOC is twice the perimeter of sector AOC . Find the value of θ correct to 2 significant figures. [5]



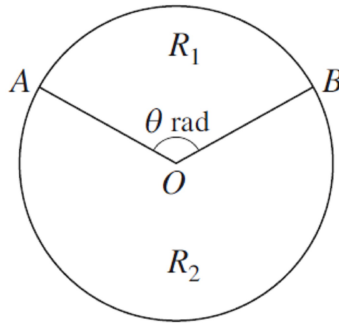
The diagram shows a circle with radius r cm and centre O . Points A and B lie on the circle and $ABCD$ is a rectangle. Angle $AOB = 2\theta$ radians and $AD = r$ cm.

- (i) Express the perimeter of the shaded region in terms of r and θ . [3]
- (ii) In the case where $r = 5$ and $\theta = \frac{1}{6}\pi$, find the area of the shaded region. [4]



The diagram shows points A and B on a circle with centre O and radius r . The tangents to the circle at A and B meet at T . The shaded region is bounded by the minor arc AB and the lines AT and BT . Angle AOB is 2θ radians.

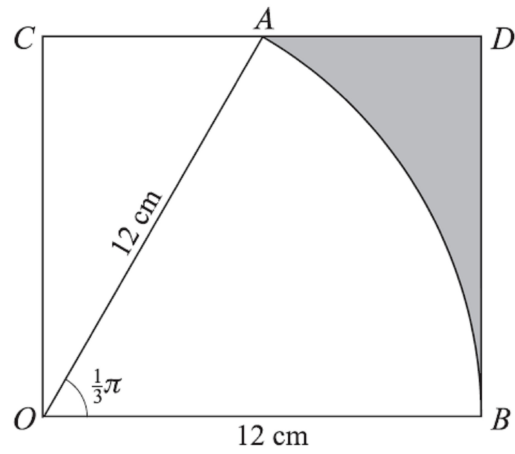
- (i) In the case where the area of the sector AOB is the same as the area of the shaded region, show that $\tan \theta = 2\theta$. [3]
- (ii) In the case where $r = 8$ cm and the length of the minor arc AB is 19.2 cm, find the area of the shaded region. [3]



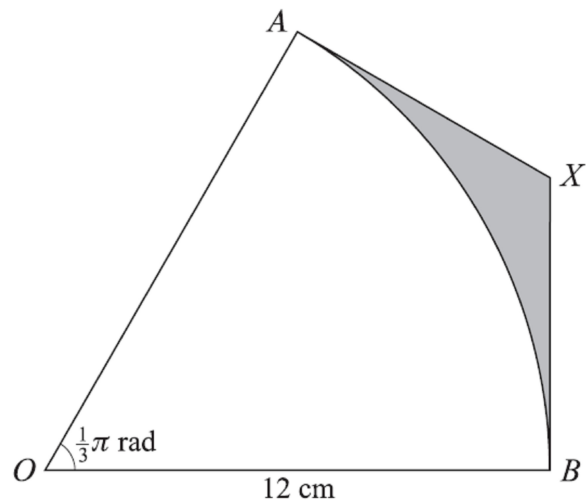
The diagram shows a circle with centre O . The circle is divided into two regions, R_1 and R_2 , by the radii OA and OB , where angle $AOB = \theta$ radians. The perimeter of the region R_1 is equal to the length of the major arc AB .

(i) Show that $\theta = \pi - 1$. [3]

(ii) Given that the area of region R_1 is 30 cm^2 , find the area of region R_2 , correct to 3 significant figures. [4]

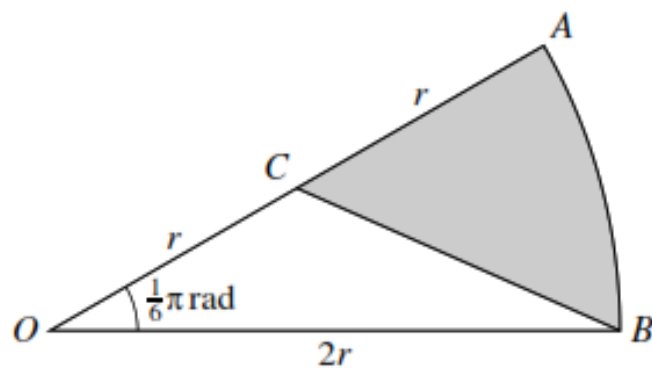


In the diagram, AOB is a sector of a circle with centre O and radius 12 cm . The point A lies on the side CD of the rectangle $OCDB$. Angle $AOB = \frac{1}{3}\pi$ radians. Express the area of the shaded region in the form $a(\sqrt{3}) - b\pi$, stating the values of the integers a and b . [6]



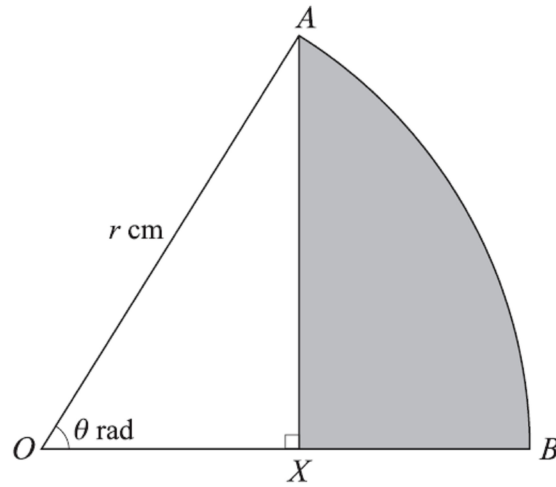
In the diagram, OAB is a sector of a circle with centre O and radius 12 cm. The lines AX and BX are tangents to the circle at A and B respectively. Angle $AOB = \frac{1}{3}\pi$ radians.

- (i) Find the exact length of AX , giving your answer in terms of $\sqrt{3}$. [2]
- (ii) Find the area of the shaded region, giving your answer in terms of π and $\sqrt{3}$. [3]



In the diagram, OAB is a sector of a circle with centre O and radius $2r$, and angle $AOB = \frac{1}{6}\pi$ radians. The point C is the midpoint of OA .

- (a) Show that the exact length of BC is $r\sqrt{5 - 2\sqrt{3}}$. [2]
- (b) Find the exact perimeter of the shaded region. [2]

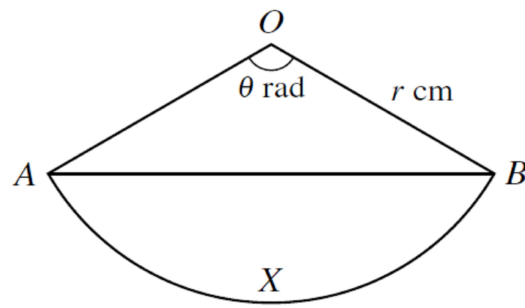


In the diagram, AB is an arc of a circle, centre O and radius r cm, and angle $AOB = \theta$ radians. The point X lies on OB and AX is perpendicular to OB .

- (i) Show that the area, A cm², of the shaded region AXB is given by

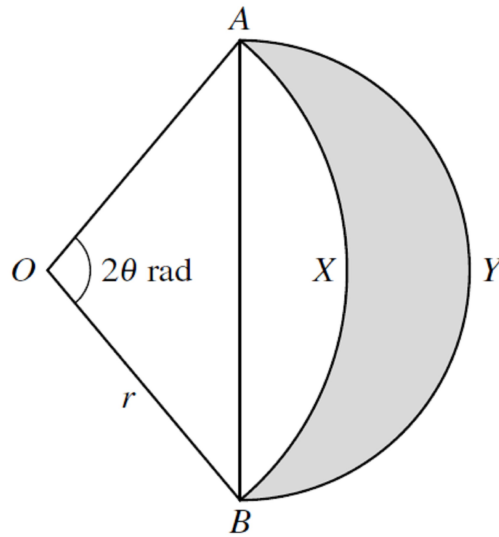
$$A = \frac{1}{2}r^2(\theta - \sin \theta \cos \theta). \quad [3]$$

- (ii) In the case where $r = 12$ and $\theta = \frac{1}{6}\pi$, find the perimeter of the shaded region AXB , leaving your answer in terms of $\sqrt{3}$ and π . [4]



The diagram shows a sector of a circle with radius r cm and centre O . The chord AB divides the sector into a triangle AOB and a segment AXB . Angle AOB is θ radians.

- (i) In the case where the areas of the triangle AOB and the segment AXB are equal, find the value of the constant p for which $\theta = p \sin \theta$. [2]
- (ii) In the case where $r = 8$ and $\theta = 2.4$, find the perimeter of the segment AXB . [3]



In the diagram, AYB is a semicircle with AB as diameter and $OAXB$ is a sector of a circle with centre O and radius r . Angle $AOB = 2\theta$ radians. Find an expression, in terms of r and θ , for the area of the shaded region. [4]