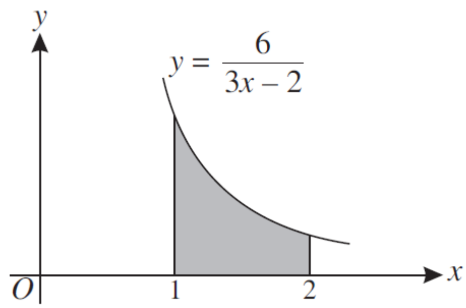
DIFFERENTIATION

M/J/2009/Q9



The diagram shows part of the curve $y = \frac{6}{3x-2}$.

(i) Find the gradient of the curve at the point where $x = 2$. [3]

M/J/2005/Q2

Find the gradient of the curve $y = \frac{12}{x^2 - 4x}$ at the point where $x = 3$. [4]

O/N/2013/Q3

The equation of a curve is $y = \dfrac{2}{\sqrt{(5x - 6)}}$.

(i) Find the gradient of the curve at the point where $x = 2$. [3]

O/N/2019/Q10

The diagram shows part of the curve $y = 1 - \dfrac{4}{(2x + 1)^2}$.

(i) Obtain expressions for $\dfrac{dy}{dx}$

M/J/2006/Q1

A curve has equation $y = \frac{k}{x}$. Given that the gradient of the curve is -3 when $x = 2$, find the value of the constant k. [3]

M/J/2011/Q4

A curve has equation $y = \frac{4}{3x - 4}$ and $P\,(2, 2)$ is a point on the curve.

(i) Find the equation of the tangent to the curve at P. [4]

(ii) Find the angle that this tangent makes with the x-axis. [2]

M/J/2010/Q10

The equation of a curve is $y = \frac{1}{6}(2x - 3)^3 - 4x$.

(i) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$. [3]

(ii) Find the equation of the tangent to the curve at the point where the curve intersects the y-axis. [3]

M/J/2005/Q9

A curve has equation $y = \dfrac{4}{\sqrt{x}}$.

(i) The normal to the curve at the point (4, 2) meets the x-axis at P and the y-axis at Q. Find the length of PQ, correct to 3 significant figures. [6]

M/J/2006/Q9

A curve is such that $\dfrac{dy}{dx} = \dfrac{4}{\sqrt{(6-2x)}}$, and $P(1, 8)$ is a point on the curve.

(i) The normal to the curve at the point P meets the coordinate axes at Q and at R. Find the coordinates of the mid-point of QR. [5]

O/N/2011/Q8

The equation of a curve is $y = \sqrt{(8x - x^2)}$. Find

(i) an expression for $\frac{dy}{dx}$, and the coordinates of the stationary point on the curve, [4]

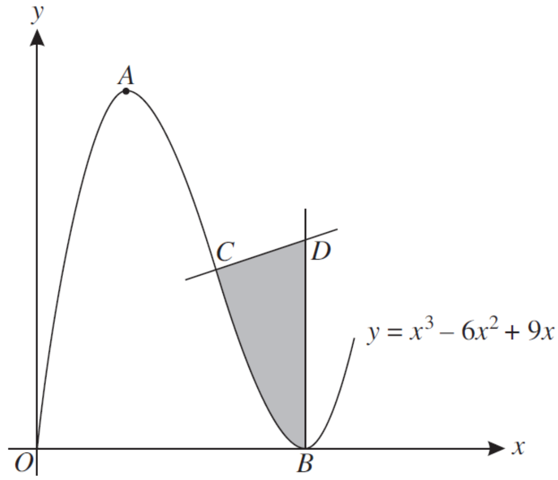
O/N/2007/O8

The equation of a curve is $y = (2x - 3)^3 - 6x$.

(i) Express $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ in terms of x. [3]

(ii) Find the x-coordinates of the two stationary points and determine the nature of each stationary point. [5]

M/J/2009/Q11



The diagram shows the curve $y = x^3 - 6x^2 + 9x$ for $x \geqslant 0$. The curve has a maximum point at A and a minimum point on the x-axis at B. The normal to the curve at $C\ (2, 2)$ meets the normal to the curve at B at the point D.

(i) Find the coordinates of A and B. [3]

(ii) Find the equation of the normal to the curve at C. [3]

M/J/2007/O10

The equation of a curve is $y = 2x + \dfrac{8}{x^2}$.

(i) Obtain expressions for $\dfrac{dy}{dx}$ and $\dfrac{d^2y}{dx^2}$. [3]

(ii) Find the coordinates of the stationary point on the curve and determine the nature of the stationary point. [3]

(iii) Show that the normal to the curve at the point $(-2, -2)$ intersects the x-axis at the point $(-10, 0)$. [3]

O/N/2017/Q8

A curve is such that $\frac{dy}{dx} = -x^2 + 5x - 4$.

(i) Find the x-coordinate of each of the stationary points of the curve. [2]

(ii) Obtain an expression for $\frac{d^2y}{dx^2}$ and hence or otherwise find the nature of each of the stationary points. [3]

M/J/2017/Q9

The equation of a curve is $y = 8\sqrt{x} - 2x$.

(i) Find the coordinates of the stationary point of the curve. [3]

(ii) Find an expression for $\frac{d^2y}{dx^2}$ and hence, or otherwise, determine the nature of the stationary point. [2]

M/J/2020/Q10

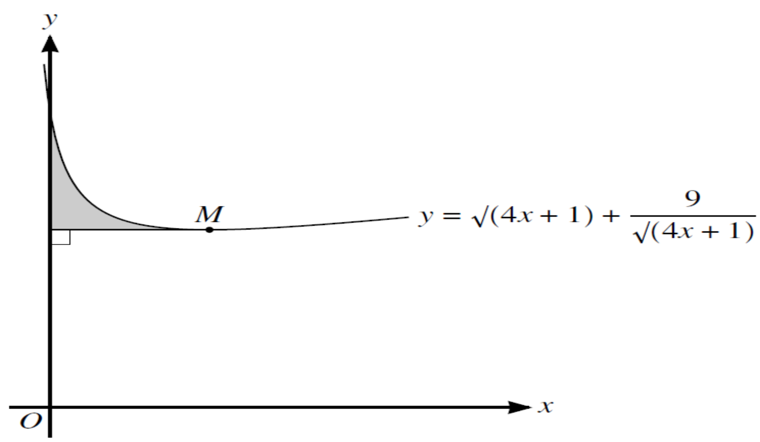
The equation of a curve is $y = 54x - (2x - 7)^3$.

(a) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. [4]

(b) Find the coordinates of each of the stationary points on the curve. [3]

(c) Determine the nature of each of the stationary points. [2]

M/J/2019/Q11

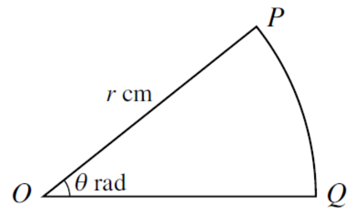


The diagram shows part of the curve $y = \sqrt{(4x + 1)} + \frac{9}{\sqrt{(4x + 1)}}$ and the minimum point M.

(i) Find expressions for $\frac{dy}{dx}$ and $\int y \, dx$. [6]

(ii) Find the coordinates of M. [3]

O/N/2009/Q7



A piece of wire of length 50 cm is bent to form the perimeter of a sector *POQ* of a circle. The radius of the circle is *r* cm and the angle *POQ* is θ radians (see diagram).

(i) Express θ in terms of r and show that the area, A cm^2, of the sector is given by

$$A = 25r - r^2.$$ [4]

(ii) Given that r can vary, find the stationary value of A and determine its nature. [4]

M/J/2010/Q8

A solid rectangular block has a square base of side x cm. The height of the block is h cm and the total surface area of the block is 96 cm^2.

(i) Express h in terms of x and show that the volume, V cm^3, of the block is given by

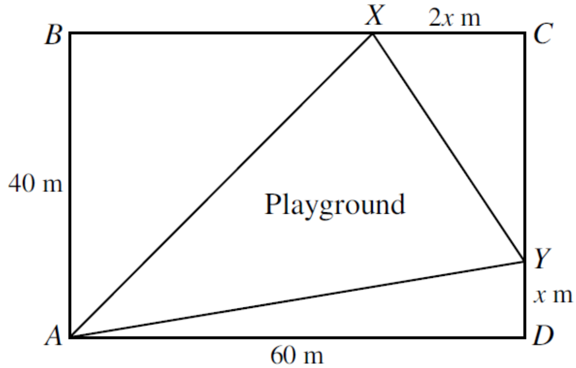
$$V = 24x - \tfrac{1}{2}x^3.$$ [3]

Given that x can vary,

(ii) find the stationary value of V, [3]

(iii) determine whether this stationary value is a maximum or a minimum. [2]

O/N/2012/Q3



The diagram shows a plan for a rectangular park $ABCD$, in which $AB = 40$ m and $AD = 60$ m. Points X and Y lie on BC and CD respectively and AX, XY and YA are paths that surround a triangular playground. The length of DY is x m and the length of XC is $2x$ m.

(i) Show that the area, A m^2, of the playground is given by

$$A = x^2 - 30x + 1200.$$ [2]

(ii) Given that x can vary, find the minimum area of the playground. [3]

M/J/2013/Q8

The volume of a solid circular cylinder of radius r cm is 250π cm^3.

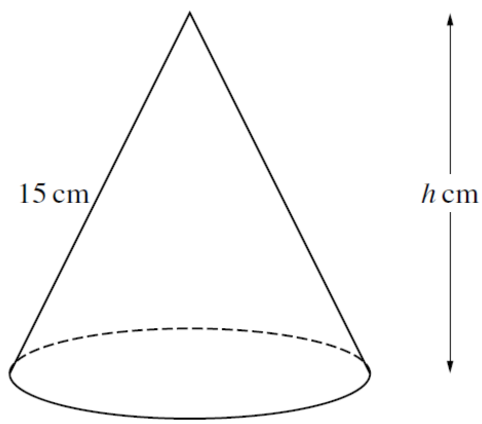
(i) Show that the total surface area, S cm^2, of the cylinder is given by

$$S = 2\pi r^2 + \frac{500\pi}{r}.$$ [2]

(ii) Given that r can vary, find the stationary value of S. [4]

(iii) Determine the nature of this stationary value. [2]

O/N/2019/Q5



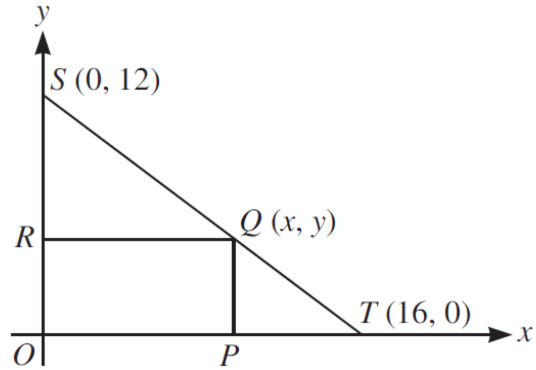
The diagram shows a solid cone which has a slant height of 15 cm and a vertical height of h cm.

(i) Show that the volume, V cm^3, of the cone is given by $V = \frac{1}{3}\pi(225h - h^3)$. [2]

[The volume of a cone of radius r and vertical height h is $\frac{1}{3}\pi r^2 h$.]

(ii) Given that h can vary, find the value of h for which V has a stationary value. Determine, showing all necessary working, the nature of this stationary value. [5]

O/N/2013/Q6

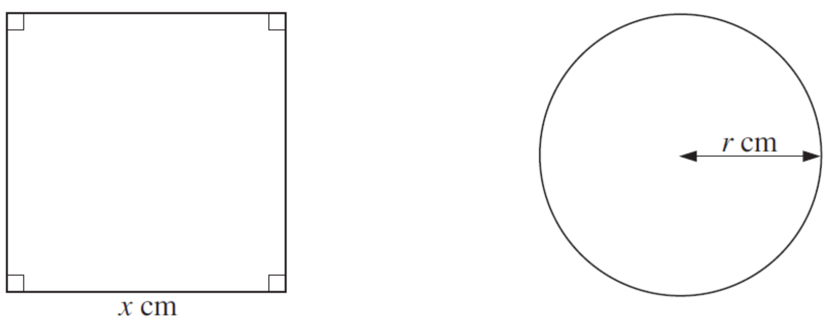


In the diagram, S is the point (0, 12) and T is the point (16, 0). The point Q lies on ST, between S and T, and has coordinates (x, y). The points P and R lie on the x-axis and y-axis respectively and $OPQR$ is a rectangle.

(i) Show that the area, A, of the rectangle $OPQR$ is given by $A = 12x - \frac{3}{4}x^2$. [3]

(ii) Given that x can vary, find the stationary value of A and determine its nature. [4]

O/N/2008/Q7

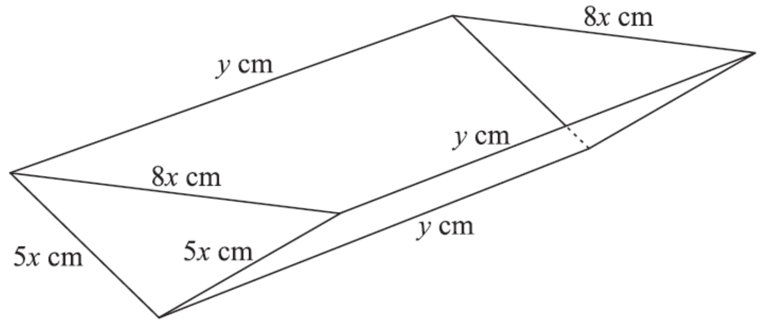


A wire, 80 cm long, is cut into two pieces. One piece is bent to form a square of side x cm and the other piece is bent to form a circle of radius r cm (see diagram). The total area of the square and the circle is A cm^2.

(i) Show that $A = \dfrac{(\pi + 4)x^2 - 160x + 1600}{\pi}$. [4]

(ii) Given that x and r can vary, find the value of x for which A has a stationary value. [4]

O/N/2006/Q9



The diagram shows an open container constructed out of $200\,\text{cm}^2$ of cardboard. The two vertical end pieces are isosceles triangles with sides $5x$ cm, $5x$ cm and $8x$ cm, and the two side pieces are rectangles of length y cm and width $5x$ cm, as shown. The open top is a horizontal rectangle.

(i) Show that $y = \dfrac{200 - 24x^2}{10x}$. [3]

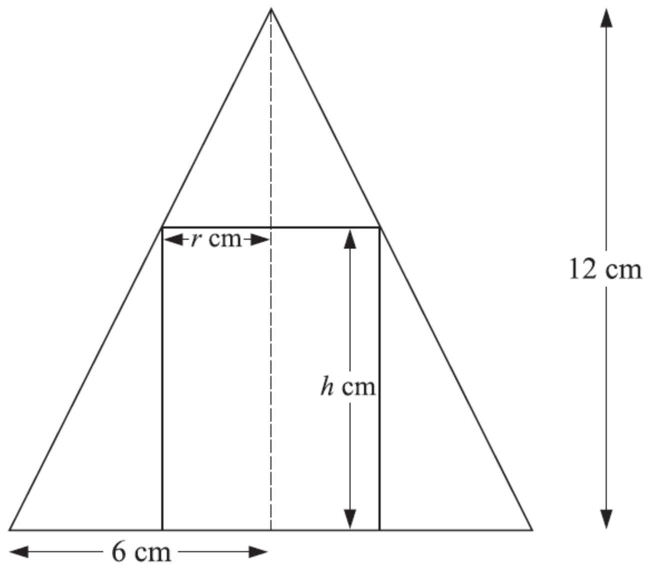
(ii) Show that the volume, $V\,\text{cm}^3$, of the container is given by $V = 240x - 28.8x^3$. [2]

Given that x can vary,

(iii) find the value of x for which V has a stationary value, [3]

(iv) determine whether it is a maximum or a minimum stationary value. [2]

O/N/2005/Q5

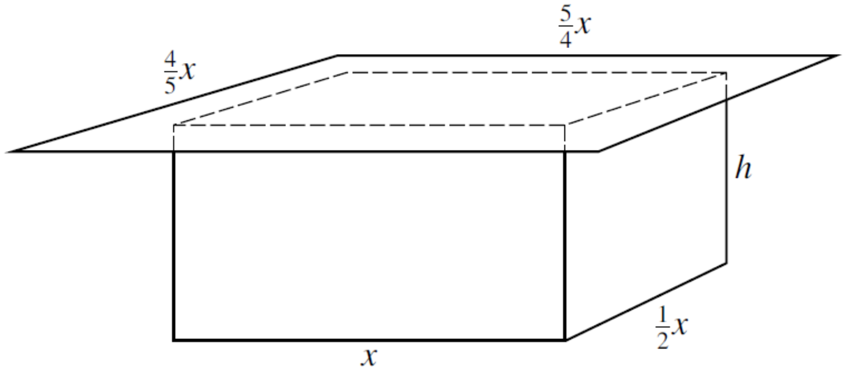


The diagram shows the cross-section of a hollow cone and a circular cylinder. The cone has radius 6 cm and height 12 cm, and the cylinder has radius r cm and height h cm. The cylinder just fits inside the cone with all of its upper edge touching the surface of the cone.

(i) Express h in terms of r and hence show that the volume, $V\,\text{cm}^3$, of the cylinder is given by

$$V = 12\pi r^2 - 2\pi r^3.$$ [3]

(ii) Given that r varies, find the stationary value of V. [4]



The diagram shows an open rectangular tank of height h metres covered with a lid. The base of the tank has sides of length x metres and $\frac{1}{2}x$ metres and the lid is a rectangle with sides of length $\frac{5}{4}x$ metres and $\frac{4}{5}x$ metres. When full the tank holds $4\,\text{m}^3$ of water. The material from which the tank is made is of negligible thickness. The external surface area of the tank together with the area of the top of the lid is $A\,\text{m}^2$.

(i) Express h in terms of x and hence show that $A = \frac{3}{2}x^2 + \dfrac{24}{x}$. [5]

(ii) Given that x can vary, find the value of x for which A is a minimum, showing clearly that A is a minimum and not a maximum. [5]

O/N/2006/Q8

The equation of a curve is $y = \dfrac{6}{5 - 2x}$.

(i) Calculate the gradient of the curve at the point where $x = 1$. [3]

(ii) A point with coordinates (x, y) moves along the curve in such a way that the rate of increase of y has a constant value of 0.02 units per second. Find the rate of increase of x when $x = 1$. [2]

M/J/2012/Q2

The equation of a curve is $y = 4\sqrt{x} + \dfrac{2}{\sqrt{x}}$.

(i) Obtain an expression for $\dfrac{dy}{dx}$. [3]

(ii) A point is moving along the curve in such a way that the x-coordinate is increasing at a constant rate of 0.12 units per second. Find the rate of change of the y-coordinate when $x = 4$. [2]

O/N/2014/Q4

A curve has equation $y = \dfrac{12}{3 - 2x}$.

(i) Find $\dfrac{dy}{dx}$. [2]

A point moves along this curve. As the point passes through A, the x-coordinate is increasing at a rate of 0.15 units per second and the y-coordinate is increasing at a rate of 0.4 units per second.

(ii) Find the possible x-coordinates of A. [4]

M/J/2019/Q3

A curve is such that $\dfrac{dy}{dx} = x^3 - \dfrac{4}{x^2}$. The point $P\,(2, 9)$ lies on the curve.

(i) A point moves on the curve in such a way that the x-coordinate is decreasing at a constant rate of 0.05 units per second. Find the rate of change of the y-coordinate when the point is at P. [2]

M/J/2017/Q5

A curve has equation $y = 3 + \frac{12}{2 - x}$.

(i) Find the equation of the tangent to the curve at the point where the curve crosses the x-axis. [5]

(ii) A point moves along the curve in such a way that the x-coordinate is increasing at a constant rate of 0.04 units per second. Find the rate of change of the y-coordinate when $x = 4$. [2]

O/N/2016/Q7

The equation of a curve is $y = 2 + \frac{3}{2x - 1}$.

(i) Obtain an expression for $\frac{dy}{dx}$. [2]

(ii) Explain why the curve has no stationary points. [1]

At the point P on the curve, $x = 2$.

(iii) Show that the normal to the curve at P passes through the origin. [4]

(iv) A point moves along the curve in such a way that its x-coordinate is decreasing at a constant rate of 0.06 units per second. Find the rate of change of the y-coordinate as the point passes through P. [2]

O/N/2015/Q3

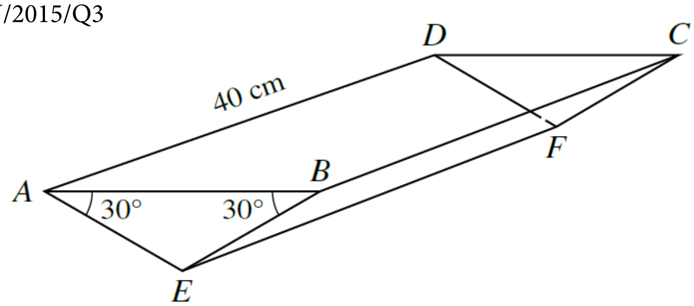


Fig. 1

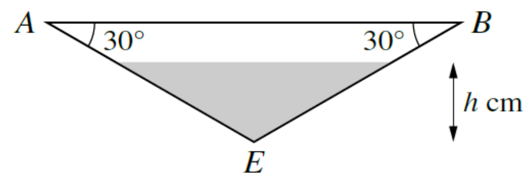


Fig. 2

Fig. 1 shows an open tank in the shape of a triangular prism. The vertical ends ABE and DCF are identical isosceles triangles. Angle ABE = angle $BAE = 30°$. The length of AD is 40 cm. The tank is fixed in position with the open top $ABCD$ horizontal. Water is poured into the tank at a constant rate of 200 $\text{cm}^3\,\text{s}^{-1}$. The depth of water, t seconds after filling starts, is h cm (see Fig. 2).

(i) Show that, when the depth of water in the tank is h cm, the volume, V cm^3, of water in the tank is given by $V = (40\sqrt{3})h^2$. [3]

(ii) Find the rate at which h is increasing when $h = 5$. [3]

M/J/2020/Q3

A weather balloon in the shape of a sphere is being inflated by a pump. The volume of the balloon is increasing at a constant rate of $600\,\text{cm}^3$ per second. The balloon was empty at the start of pumping.

(a) Find the radius of the balloon after 30 seconds. [2]

(b) Find the rate of increase of the radius after 30 seconds. [3]