DIFFERENTIATION

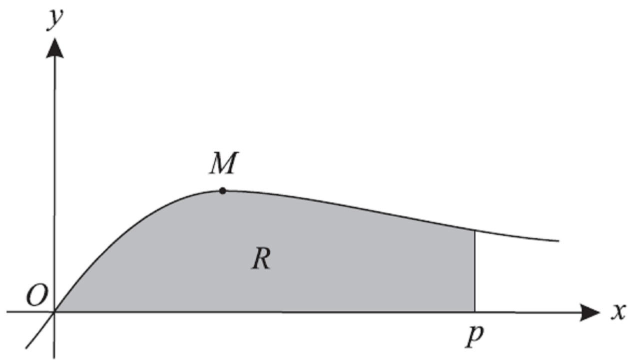
Friday, 26 August 2022 10:11 AM

O/N/2013/Q1

The equation of a curve is $y = \dfrac{1+x}{1+2x}$ for $x > -\frac{1}{2}$. Show that the gradient of the curve is always negative. [3]

M/J/2005/Q9

M.JAWAD AMIN



The diagram shows part of the curve $y = \dfrac{x}{x^2 + 1}$ and its maximum point M. The shaded region R is bounded by the curve and by the lines $y = 0$ and $x = p$.

(i) Calculate the x-coordinate of M. [4]

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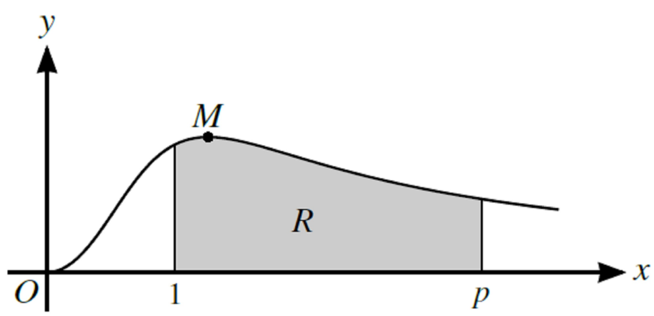
O/N/2015/Q10



The diagram shows the curve $y = \dfrac{x^2}{1 + x^3}$ for $x \geqslant 0$, and its maximum point M. The shaded region R is enclosed by the curve, the x-axis and the lines $x = 1$ and $x = p$.

(i) Find the exact value of the x-coordinate of M. [4]

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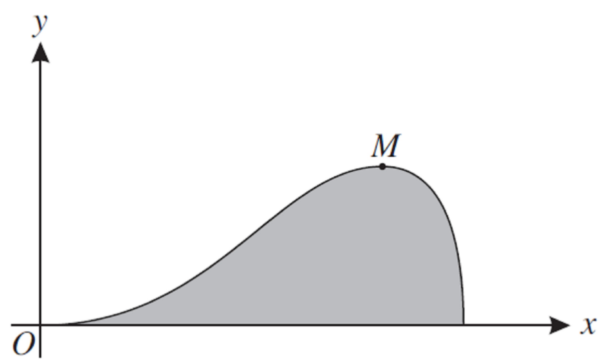
M/J/2009/Q10



The diagram shows the curve $y = x^2\sqrt{(1 - x^2)}$ for $x \geqslant 0$ and its maximum point M.

(i) Find the exact value of the x-coordinate of M. [4]

M.JAWAD AMIN

O/N/2009/Q3

The equation of a curve is $x^3 - x^2y - y^3 = 3$.

(i) Find $\frac{dy}{dx}$ in terms of x and y. [4]

(ii) Find the equation of the tangent to the curve at the point (2, 1), giving your answer in the form $ax + by + c = 0$. [2]

M.JAWAD AMIN

M/J/2018/Q5

The equation of a curve is $x^2(x + 3y) - y^3 = 3$.

(i) Show that $\frac{dy}{dx} = \frac{x^2 + 2xy}{y^2 - x^2}$. [4]

(ii) Hence find the exact coordinates of the two points on the curve at which the gradient of the normal is 1. [4]

O/N/2019/Q5

The equation of a curve is $2x^2y - xy^2 = a^3$, where a is a positive constant. Show that there is only one point on the curve at which the tangent is parallel to the x-axis and find the y-coordinate of this point. [7]

M.JAWAD AMIN

M/J/2008/Q6

The equation of a curve is $xy(x + y) = 2a^3$, where a is a non-zero constant. Show that there is only one point on the curve at which the tangent is parallel to the x-axis, and find the coordinates of this point. [8]

O/N/2012/Q7

The equation of a curve is $\ln(xy) - y^3 = 1$.

(i) Show that $\frac{dy}{dx} = \frac{y}{x(3y^3 - 1)}$. [4]

(ii) Find the coordinates of the point where the tangent to the curve is parallel to the y-axis, giving each coordinate correct to 3 significant figures. [4]

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M.JAWAD AMIN

M/J/2010/Q6

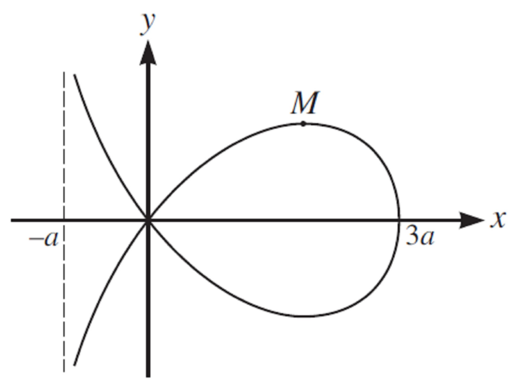
The equation of a curve is

$$x \ln y = 2x + 1.$$

(i) Show that $\frac{dy}{dx} = -\frac{y}{x^2}$. [4]

(ii) Find the equation of the tangent to the curve at the point where $y = 1$, giving your answer in the form $ax + by + c = 0$. [4]

M/J/2013/Q5



The diagram shows the curve with equation

$$x^3 + xy^2 + ay^2 - 3ax^2 = 0,$$

where a is a positive constant. The maximum point on the curve is M. Find the x-coordinate of M in terms of a. [6]

M.JAWAD AMIN

O/N/2006/Q3

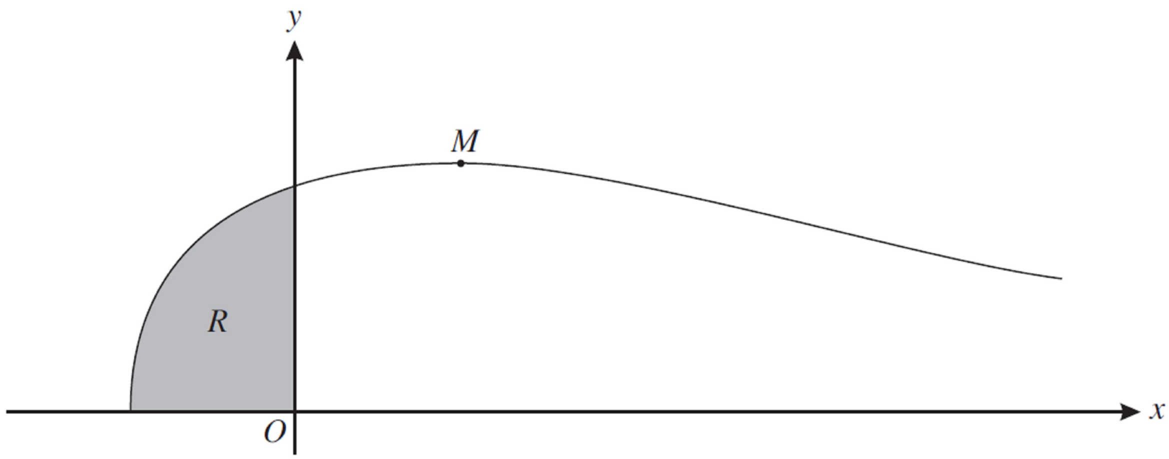
The curve with equation $y = 6e^x - e^{3x}$ has one stationary point.

(i) Find the x-coordinate of this point. [4]

(ii) Determine whether this point is a maximum or a minimum point. [2]

M.JAWAD AMIN

M/J/2008/Q9



The diagram shows the curve $y = e^{-\frac{1}{2}x}\sqrt{(1 + 2x)}$ and its maximum point M. The shaded region between the curve and the axes is denoted by R.

(i) Find the x-coordinate of M. [4]

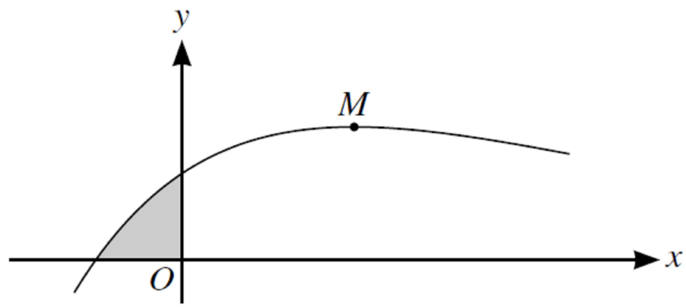
M.JAWAD AMIN

O/N/2019/Q2

The curve with equation $y = \dfrac{e^{-2x}}{1 - x^2}$ has a stationary point in the interval $-1 < x < 1$. Find $\dfrac{dy}{dx}$ and hence find the x-coordinate of this stationary point, giving the answer correct to 3 decimal places. [5]

M.JAWAD AMIN

M/J/2018/Q8



The diagram shows the curve $y = (x + 1)e^{-\frac{1}{3}x}$ and its maximum point M.

(i) Find the x-coordinate of M. [4]

M.JAWAD AMIN

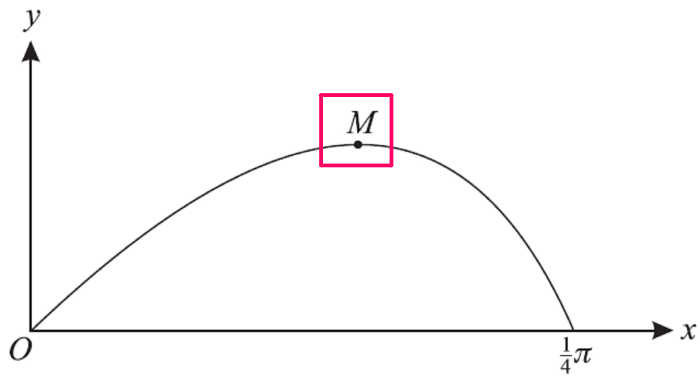
M/J/2019/Q4

Find the exact coordinates of the point on the curve $y = \dfrac{x}{1 + \ln x}$ at which the gradient of the tangent is equal to $\frac{1}{4}$. [7]

O/N/2005/Q3

The equation of a curve is $y = x + \cos 2x$. Find the x-coordinates of the stationary points of the curve for which $0 \leqslant x \leqslant \pi$, and determine the nature of each of these stationary points. [7]

M.JAWAD AMIN



The diagram shows the curve $y = x\cos 2x$ for $0 \leqslant x \leqslant \frac{1}{4}\pi$. The point M is a maximum point.

(i) Show that the x-coordinate of M satisfies the equation $1 = 2x\tan 2x$. [3]

M/J/2007/Q3

The equation of a curve is $y = x \sin 2x$, where x is in radians. Find the equation of the tangent to the curve at the point where $x = \frac{1}{4}\pi$. [4]

M.JAWAD AMIN

O/N/2007/Q4

The curve with equation $y = e^{-x} \sin x$ has one stationary point for which $0 \leqslant x \leqslant \pi$.

(i) Find the x-coordinate of this point. [4]

(ii) Determine whether this point is a maximum or a minimum point. [2]

M.JAWAD AMIN

M/J/2015/Q3

A curve has equation $y = \cos x \cos 2x$. Find the x-coordinate of the stationary point on the curve in the interval $0 < x < \frac{1}{2}\pi$, giving your answer correct to 3 significant figures. [6]

M.JAWAD AMIN

O/N/2017/Q4

The curve with equation $y = \dfrac{2 - \sin x}{\cos x}$ has one stationary point in the interval $-\frac{1}{2}\pi < x < \frac{1}{2}\pi$.

(i) Find the exact coordinates of this point. [5]

(ii) Determine whether this point is a maximum or a minimum point. [2]

M.JAWAD AMIN

O/N/2018/Q7

A curve has equation $y = \dfrac{3\cos x}{2 + \sin x}$, for $-\frac{1}{2}\pi \leqslant x \leqslant \frac{1}{2}\pi$.

(i) Find the exact coordinates of the stationary point of the curve. [6]

O/N/2012/Q5

(i) By differentiating $\frac{1}{\cos x}$, show that if $y = \sec x$ then $\frac{dy}{dx} = \sec x \tan x$. [2]

(ii) Show that $\frac{1}{\sec x - \tan x} \equiv \sec x + \tan x$. [1]

M.JAWAD AMIN

M/J/2012/Q6

The equation of a curve is $y = 3\sin x + 4\cos^3 x$.

(i) Find the x-coordinates of the stationary points of the curve in the interval $0 < x < \pi$. [6]

(ii) Determine the nature of the stationary point in this interval for which x is least. [2]

O/N/2015/Q5

The equation of a curve is $y = e^{-2x} \tan x$, for $0 \leqslant x < \frac{1}{2}\pi$.

(i) Obtain an expression for $\frac{dy}{dx}$ and show that it can be written in the form $e^{-2x}(a + b\tan x)^2$, where a and b are constants. [5]

(ii) Explain why the gradient of the curve is never negative. [1]

(iii) Find the value of x for which the gradient is least. [1]

M.JAWAD AMIN

M/J/2006/Q3

The parametric equations of a curve are

$$x = 2\theta + \sin 2\theta, \quad y = 1 - \cos 2\theta.$$

Show that $\frac{dy}{dx} = \tan\theta$. [5]

M.JAWAD AMIN

O/N/2008/Q4

The parametric equations of a curve are

$$x = a(2\theta - \sin 2\theta), \qquad y = a(1 - \cos 2\theta).$$

Show that $\frac{dy}{dx} = \cot\theta$. [5]

M.JAWAD AMIN

M/J/2017/Q4

The parametric equations of a curve are

$$x = t^2 + 1, \quad y = 4t + \ln(2t - 1).$$

(i) Express $\dfrac{dy}{dx}$ in terms of t. [3]

(ii) Find the equation of the normal to the curve at the point where $t = 1$. Give your answer in the form $ax + by + c = 0$. [3]

11)

The parametric equations of a curve are

$$x = t - \tan t, \quad y = \ln(\cos t),$$

for $-\frac{1}{2}\pi < t < \frac{1}{2}\pi$.

(i) Show that $\dfrac{dy}{dx} = \cot t$. [5]

(ii) Hence find the x-coordinate of the point on the curve at which the gradient is equal to 2. Give your answer correct to 3 significant figures. [2]

The parametric equations of a curve are

$$x = \ln(\tan t), \quad y = \sin^2 t,$$

where $0 < t < \frac{1}{2}\pi$.

(i) Express $\frac{dy}{dx}$ in terms of t. [4]

(ii) Find the equation of the tangent to the curve at the point where $x = 0$. [3]

M/J/2009/Q6

The parametric equations of a curve are

$$x = a\cos^3 t, \quad y = a\sin^3 t,$$

where a is a positive constant and $0 < t < \frac{1}{2}\pi$.

(i) Express $\frac{dy}{dx}$ in terms of t. [3]

(ii) Show that the equation of the tangent to the curve at the point with parameter t is

$$x\sin t + y\cos t = a\sin t\cos t.$$ [3]

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The parametric equations of a curve are

$$x = 3(1 + \sin^2 t), \quad y = 2\cos^3 t.$$

Find $\frac{dy}{dx}$ in terms of t, simplifying your answer as far as possible. [5]

M.JAWAD AMIN

O/N/2014/Q4

The parametric equations of a curve are

$$x = \frac{1}{\cos^3 t}, \quad y = \tan^3 t,$$

where $0 \leqslant t < \frac{1}{2}\pi$.

(i) Show that $\frac{dy}{dx} = \sin t$. [4]

(ii) Hence show that the equation of the tangent to the curve at the point with parameter t is $y = x \sin t - \tan t$. [3]

M.JAWAD AMIN

The parametric equations of a curve are

$$x = e^{-t} \cos t, \quad y = e^{-t} \sin t.$$

Show that $\frac{dy}{dx} = \tan\left(t - \frac{1}{4}\pi\right)$. [6]

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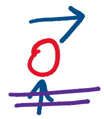

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