O/N/2006/Q8

Let $f(x) = \dfrac{7x + 4}{(2x + 1)(x + 1)^2}$.

(i) Express $f(x)$ in partial fractions. [5]

(ii) Hence show that $\int_0^2 f(x)\,dx = 2 + \ln\frac{5}{3}$. [5]

Let $f(x) = \dfrac{10x + 9}{(2x + 1)(2x + 3)^2}$.

(i) Express $f(x)$ in partial fractions. [5]

(ii) Hence show that $\int_0^1 f(x)\,dx = \frac{1}{2}\ln\frac{9}{5} + \frac{1}{5}$. [5]

Let $f(x) = \dfrac{2x^2 + x + 8}{(2x - 1)(x^2 + 2)}$.

(i) Express $f(x)$ in partial fractions. [5]

(ii) Hence, showing full working, find $\int_1^5 f(x)\,dx$, giving the answer in the form $\ln c$, where c is an integer. [5]

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Let $f(x) = \dfrac{12 + 8x - x^2}{(2 - x)(4 + x^2)}$.

(i) Express $f(x)$ in the form $\dfrac{A}{2 - x} + \dfrac{Bx + C}{4 + x^2}$. [4]

(ii) Show that $\displaystyle\int_0^1 f(x)\,dx = \ln\left(\tfrac{25}{2}\right)$. [5]

Let $f(x) \equiv \dfrac{x^2 + 3x + 3}{(x+1)(x+3)}$.

(i) Express $f(x)$ in partial fractions. [5]

(ii) Hence show that $\int_0^3 f(x)\,dx = 3 - \frac{1}{2}\ln 2$. [4]

(i) Find the values of the constants A, B, C and D such that

$$\frac{2x^3 - 1}{x^2(2x - 1)} \equiv A + \frac{B}{x} + \frac{C}{x^2} + \frac{D}{2x - 1}.$$ [5]

(ii) Hence show that

$$\int_1^2 \frac{2x^3 - 1}{x^2(2x - 1)}\,\mathrm{d}x = \tfrac{3}{2} + \tfrac{1}{2}\ln\left(\tfrac{16}{27}\right).$$ [5]

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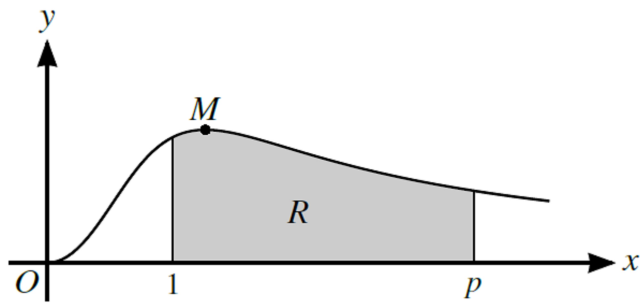

The diagram shows the curve $y = \frac{x^2}{1+x^3}$ for $x \geqslant 0$, and its maximum point M. The shaded region R is enclosed by the curve, the x-axis and the lines $x = 1$ and $x = p$.

(i) Find the exact value of the x-coordinate of M. [4]

(ii) Calculate the value of p for which the area of R is equal to 1. Give your answer correct to 3 significant figures. [6]

Let $I = \int_1^4 \frac{1}{x(4-\sqrt{x})}\,dx$.

(i) Use the substitution $u = \sqrt{x}$ to show that $I = \int_1^2 \frac{2}{u(4-u)}\,du$. [3]

(ii) Hence show that $I = \frac{1}{2}\ln 3$. [6]

Let $I = \displaystyle\int_0^1 \frac{\sqrt{x}}{2 - \sqrt{x}}\,dx$.

(i) Using the substitution $u = 2 - \sqrt{x}$, show that $I = \displaystyle\int_1^2 \frac{2(2-u)^2}{u}\,du$. [4]

(ii) Hence show that $I = 8\ln 2 - 5$. [4]

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Let $I = \int_2^5 \frac{5}{x + \sqrt{(6 - x)}}\, dx$.

(i) Using the substitution $u = \sqrt{(6 - x)}$, show that

$$I = \int_1^2 \frac{10u}{(3 - u)(2 + u)}\, du. \qquad [4]$$

(ii) Hence show that $I = 2\ln\left(\frac{9}{2}\right)$. [6]

(i) Find $\int \frac{\ln x}{x^3}\,dx$. [3]

(ii) Hence show that $\int_1^2 \frac{\ln x}{x^3}\,dx = \frac{1}{16}(3 - \ln 4)$. [2]

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O/N/2013/Q3

Find the exact value of $\displaystyle\int_1^4 \frac{\ln x}{\sqrt{x}}\,dx$. [5]

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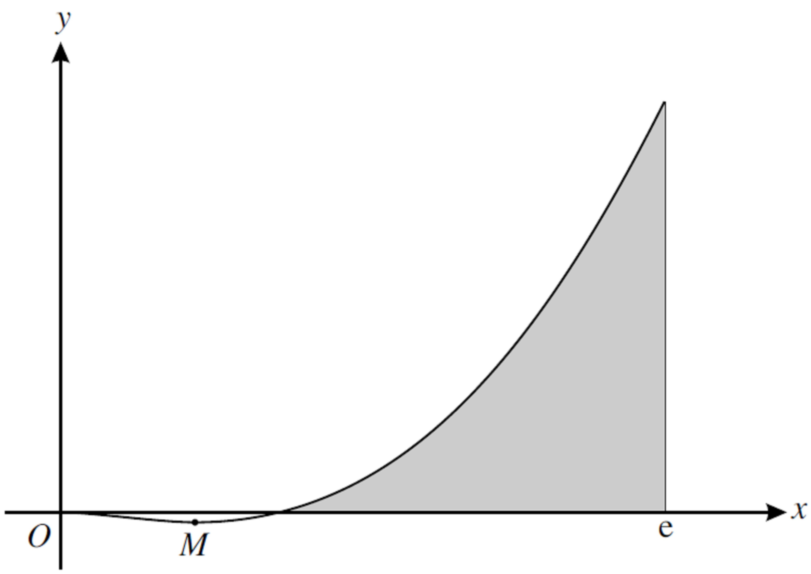

The diagram shows the curve $y = x^2 \ln x$ and its minimum point M.

(i) Find the exact values of the coordinates of M. [5]

(ii) Find the exact value of the area of the shaded region bounded by the curve, the x-axis and the line $x = \text{e}$. [5]

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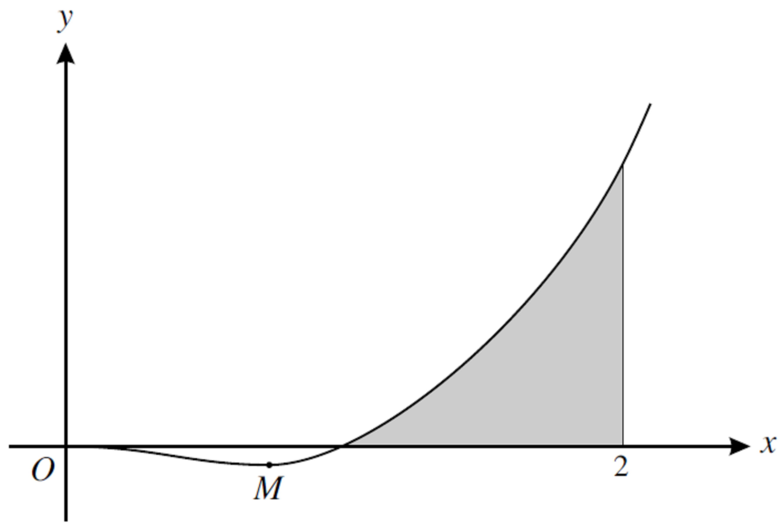
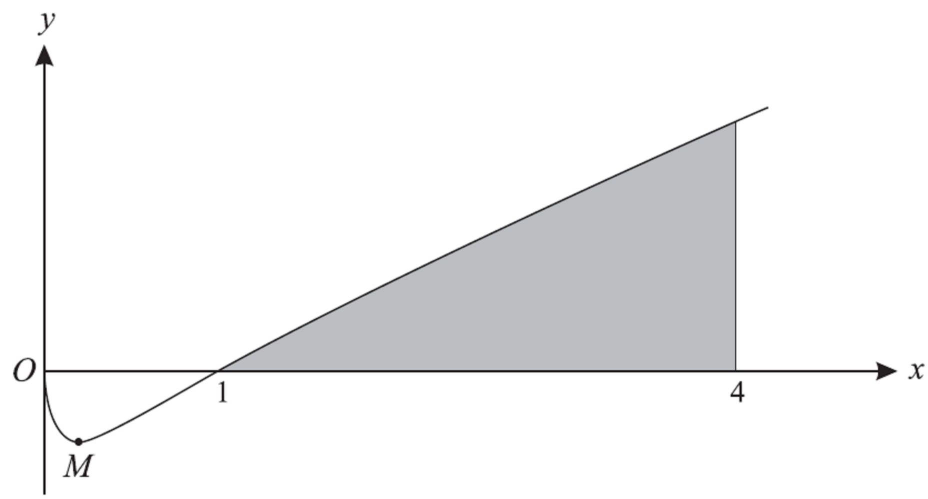


The diagram shows the curve $y = x^3 \ln x$ and its minimum point M.

(i) Find the exact coordinates of M. [5]

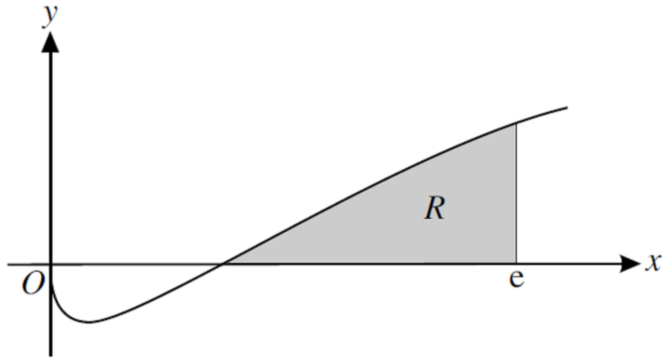
(ii) Find the exact area of the shaded region bounded by the curve, the x-axis and the line $x = 2$. [5]



The diagram shows a sketch of the curve $y = x^{\frac{1}{2}} \ln x$ and its minimum point M. The curve cuts the x-axis at the point $(1, 0)$.

(i) Find the exact value of the x-coordinate of M. [4]

(ii) Use integration by parts to find the area of the shaded region enclosed by the curve, the x-axis and the line $x = 4$. Give your answer correct to 2 decimal places. [5]



The diagram shows the curve $y = x^{\frac{1}{2}} \ln x$. The shaded region between the curve, the x-axis and the line $x = \mathrm{e}$ is denoted by R.

(i) Find the equation of the tangent to the curve at the point where $x = 1$, giving your answer in the form $y = mx + c$. [4]

(ii) Find by integration the volume of the solid obtained when the region R is rotated completely about the x-axis. Give your answer in terms of π and e. [7]

O/N/2007/Q3

Use integration by parts to show that

$$\int_2^4 \ln x \, dx = 6 \ln 2 - 2. \qquad [4]$$

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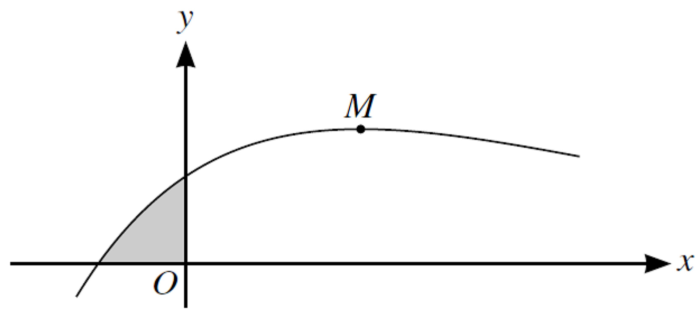
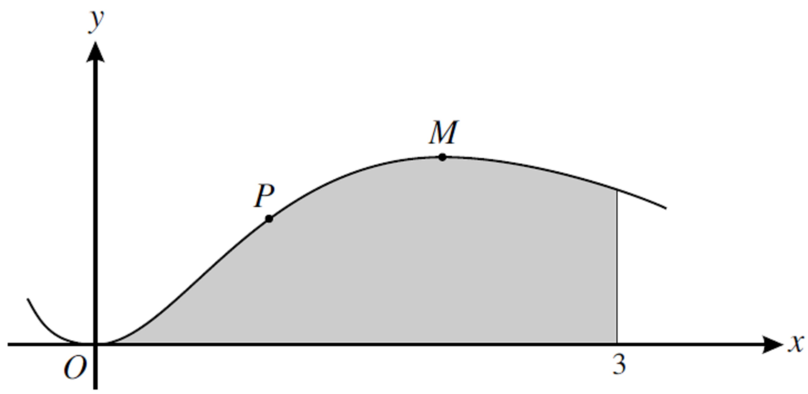


The diagram shows the curve $y = (x + 1)e^{-\frac{1}{3}x}$ and its maximum point M.

(i) Find the x-coordinate of M. [4]

(ii) Find the area of the shaded region enclosed by the curve and the axes, giving your answer in terms of e. [5]

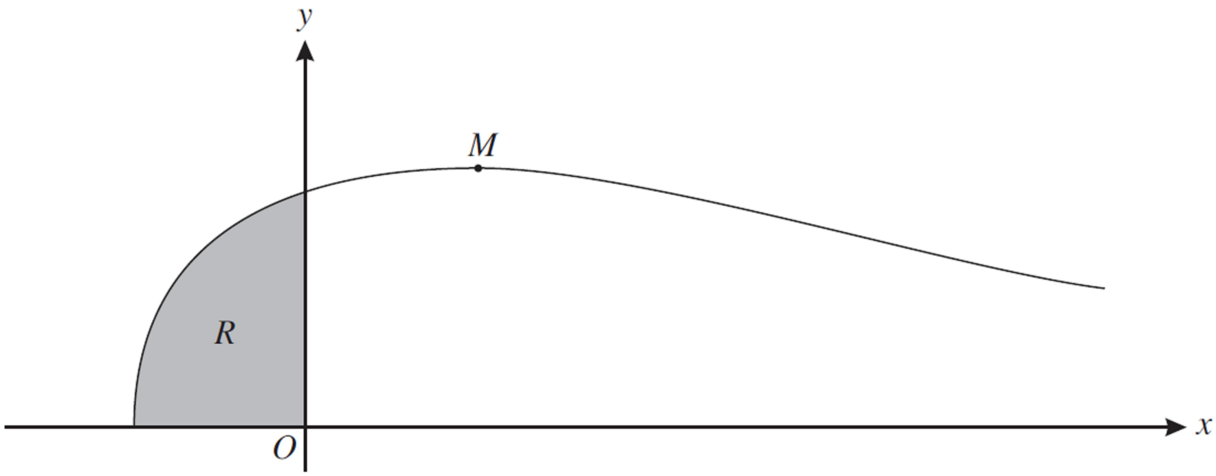


The diagram shows the curve $y = x^2e^{-x}$.

(i) Show that the area of the shaded region bounded by the curve, the x-axis and the line $x = 3$ is equal to $2 - \frac{17}{e^3}$. [5]

(ii) Find the x-coordinate of the maximum point M on the curve. [4]

(iii) Find the x-coordinate of the point P at which the tangent to the curve passes through the origin. [2]

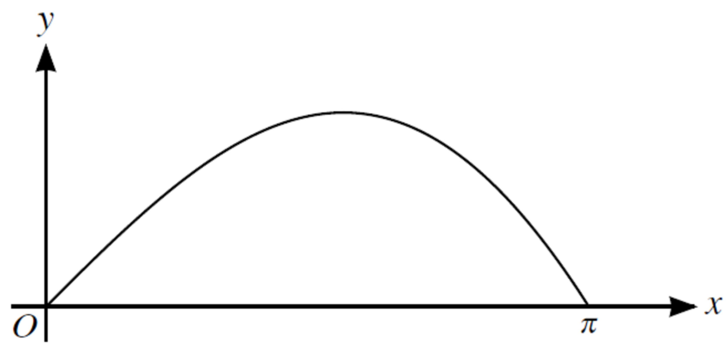


The diagram shows the curve $y = e^{-\frac{1}{2}x}\sqrt{(1 + 2x)}$ and its maximum point M. The shaded region between the curve and the axes is denoted by R.

(i) Find the x-coordinate of M. [4]

(ii) Find by integration the volume of the solid obtained when R is rotated completely about the x-axis. Give your answer in terms of π and e. [6]

Show that $\int_0^{\pi} x^2 \sin x \, dx = \pi^2 - 4.$ [5]



The diagram shows the curve $y = x\cos\frac{1}{2}x$ for $0 \leqslant x \leqslant \pi$.

(i) Find $\frac{dy}{dx}$ and show that $4\frac{d^2y}{dx^2} + y + 4\sin\frac{1}{2}x = 0$. [5]

(ii) Find the exact value of the area of the region enclosed by this part of the curve and the x-axis. [5]

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(i) Prove that $\cot\theta + \tan\theta \equiv 2\operatorname{cosec} 2\theta$. [3]

(ii) Hence show that $\int_{\frac{1}{6}\pi}^{\frac{1}{3}\pi} \operatorname{cosec} 2\theta \, d\theta = \frac{1}{2}\ln 3$. [4]

(i) Show that $\dfrac{2\sin x - \sin 2x}{1 - \cos 2x} \equiv \dfrac{\sin x}{1 + \cos x}$. [4]

(ii) Hence, showing all necessary working, find $\displaystyle\int_{\frac{1}{3}\pi}^{\frac{1}{2}\pi} \frac{2\sin x - \sin 2x}{1 - \cos 2x}\,dx$, giving your answer in the form $\ln k$. [4]

(i) Prove that if $y = \dfrac{1}{\cos\theta}$ then $\dfrac{dy}{d\theta} = \sec\theta\tan\theta$. [2]

(ii) Prove the identity $\dfrac{1+\sin\theta}{1-\sin\theta} \equiv 2\sec^2\theta + 2\sec\theta\tan\theta - 1$. [3]

(iii) Hence find the exact value of $\displaystyle\int_0^{\frac{1}{4}\pi} \frac{1+\sin\theta}{1-\sin\theta}\,d\theta$. [4]

(i) By differentiating $\frac{1}{\cos x}$, show that if $y = \sec x$ then $\frac{dy}{dx} = \sec x \tan x$. [2]

(ii) Show that $\frac{1}{\sec x - \tan x} \equiv \sec x + \tan x$. [1]

(iii) Deduce that $\frac{1}{(\sec x - \tan x)^2} \equiv 2\sec^2 x - 1 + 2\sec x \tan x$. [2]

(iv) Hence show that $\int_0^{\frac{1}{4}\pi} \frac{1}{(\sec x - \tan x)^2}\, dx = \frac{1}{4}(8\sqrt{2} - \pi)$. [3]

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(i) Use the substitution $x = \tan\theta$ to show that

$$\int \frac{1-x^2}{(1+x^2)^2}\,\mathrm{d}x = \int \cos 2\theta\,\mathrm{d}\theta.$$ [4]

(ii) Hence find the value of

$$\int_0^1 \frac{1-x^2}{(1+x^2)^2}\,\mathrm{d}x.$$ [3]

(i) Use the substitution $x = 2\tan\theta$ to show that

$$\int_0^2 \frac{8}{(4+x^2)^2}\,dx = \int_0^{\frac{1}{4}\pi} \cos^2\theta\,d\theta. \qquad [4]$$

(ii) Hence find the exact value of

$$\int_0^2 \frac{8}{(4+x^2)^2}\,dx. \qquad [4]$$

O/N/2010/Q5

Let $I = \int_0^1 \frac{x^2}{\sqrt{(4 - x^2)}}\,dx$.

(i) Using the substitution $x = 2\sin\theta$, show that

$$I = \int_0^{\frac{1}{6}\pi} 4\sin^2\theta\,d\theta. \qquad [3]$$

(ii) Hence find the exact value of I. [4]

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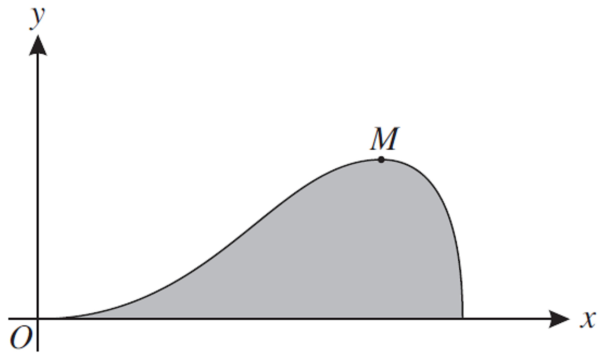

The diagram shows the curve $y = x^2\sqrt{(1 - x^2)}$ for $x \geqslant 0$ and its maximum point M.

(i) Find the exact value of the x-coordinate of M. [4]

(ii) Show, by means of the substitution $x = \sin\theta$, that the area A of the shaded region between the curve and the x-axis is given by

$$A = \frac{1}{4}\int_0^{\frac{1}{2}\pi} \sin^2 2\theta \, d\theta.$$ [3]

(iii) Hence obtain the exact value of A. [4]

(i) By differentiating $\frac{1}{\cos x}$, show that the derivative of $\sec x$ is $\sec x \tan x$. Hence show that if $y = \ln(\sec x + \tan x)$ then $\frac{\mathrm{d}y}{\mathrm{d}x} = \sec x$. [4]

(ii) Using the substitution $x = (\sqrt{3}) \tan \theta$, find the exact value of

$$\int_1^3 \frac{1}{\sqrt{(3 + x^2)}} \, \mathrm{d}x,$$

expressing your answer as a single logarithm. [4]

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