QUADRATICAL FUNTIONS

O/N/2007/Q11

The function f is defined by $f : x \mapsto 2x^2 - 8x + 11$ for $x \in \mathbb{R}$.

(i) Express $f(x)$ in the form $a(x+b)^2 + c$, where a, b and c are constants. [3]

(ii) State the range of f. [1]

(iii) Explain why f does not have an inverse. [1]

The function g is defined by $g : x \mapsto 2x^2 - 8x + 11$ for $x \leqslant A$, where A is a constant.

(iv) State the largest value of A for which g has an inverse. [1]

(v) When A has this value, obtain an expression, in terms of x, for $g^{-1}(x)$ and state the range of g^{-1}. [4]

M.JAWAD AMIN

O/N/2018/Q9

The function f is defined by $f : x \mapsto 2x^2 - 12x + 7$ for $x \in \mathbb{R}$.

(i) Express $2x^2 - 12x + 7$ in the form $2(x + a)^2 + b$, where a and b are constants. [2]

(ii) State the range of f. [1]

The function g is defined by $g : x \mapsto 2x^2 - 12x + 7$ for $x \leqslant k$.

(iii) State the largest value of k for which g has an inverse. [1]

(iv) Given that g has an inverse, find an expression for $g^{-1}(x)$. [3]

M.JAWAD AMIN

M/J/2018/Q7

The function f is defined by $f : x \mapsto 7 - 2x^2 - 12x$ for $x \in \mathbb{R}$.

(i) Express $7 - 2x^2 - 12x$ in the form $a - 2(x + b)^2$, where a and b are constants. [2]

(ii) State the coordinates of the stationary point on the curve $y = f(x)$. [1]

The function g is defined by $g : x \mapsto 7 - 2x^2 - 12x$ for $x \geqslant k$.

(iii) State the smallest value of k for which g has an inverse. [1]

(iv) For this value of k, find $g^{-1}(x)$. [3]

M.JAWAD AMIN

O/N/2010/Q7

The function f is defined by

$$f(x) = x^2 - 4x + 7 \quad \text{for} \quad x > 2.$$

(i) Express $f(x)$ in the form $(x - a)^2 + b$ and hence state the range of f. [3]

(ii) Obtain an expression for $f^{-1}(x)$ and state the domain of f^{-1}. [3]

The function g is defined by

$$g(x) = x - 2 \quad \text{for} \quad x > 2.$$

The function h is such that f = hg and the domain of h is $x > 0$.

(iii) Obtain an expression for $h(x)$. [1]

M/J/2015/Q11

The function f is defined by $f : x \mapsto 2x^2 - 6x + 5$ for $x \in \mathbb{R}$.

(i) Find the set of values of p for which the equation $f(x) = p$ has no real roots. [3]

The function g is defined by $g : x \mapsto 2x^2 - 6x + 5$ for $0 \leqslant x \leqslant 4$.

(ii) Express $g(x)$ in the form $a(x + b)^2 + c$, where a, b and c are constants. [3]

(iii) Find the range of g. [2]

The function h is defined by $h : x \mapsto 2x^2 - 6x + 5$ for $k \leqslant x \leqslant 4$, where k is a constant.

(iv) State the smallest value of k for which h has an inverse. [1]

(v) For this value of k, find an expression for $h^{-1}(x)$. [3]

M/J/2009/Q10

The function f is defined by $f : x \mapsto 2x^2 - 12x + 13$ for $0 \leqslant x \leqslant A$, where A is a constant.

(i) Express $f(x)$ in the form $a(x + b)^2 + c$, where a, b and c are constants. [3]

(ii) State the value of A for which the graph of $y = f(x)$ has a line of symmetry. [1]

(iii) When A has this value, find the range of f. [2]

The function g is defined by $g : x \mapsto 2x^2 - 12x + 13$ for $x \geqslant 4$.

(iv) Explain why g has an inverse. [1]

(v) Obtain an expression, in terms of x, for $g^{-1}(x)$. [3]

M.JAWAD AMIN

M/J/2016/Q11

The function f is defined by $f : x \mapsto 6x - x^2 - 5$ for $x \in \mathbb{R}$.

(i) Find the set of values of x for which $f(x) \leqslant 3$. [3]

(ii) Given that the line $y = mx + c$ is a tangent to the curve $y = f(x)$, show that $4c = m^2 - 12m + 16$. [3]

The function g is defined by $g : x \mapsto 6x - x^2 - 5$ for $x \geqslant k$, where k is a constant.

(iii) Express $6x - x^2 - 5$ in the form $a - (x - b)^2$, where a and b are constants. [2]

(iv) State the smallest value of k for which g has an inverse. [1]

(v) For this value of k, find an expression for $g^{-1}(x)$. [2]

O/N/2019/Q9

Functions f and g are defined by

$$f(x) = 2x^2 + 8x + 1 \quad \text{for } x \in \mathbb{R},$$
$$g(x) = 2x - k \quad \text{for } x \in \mathbb{R},$$

where k is a constant.

(i) Find the value of k for which the line $y = g(x)$ is a tangent to the curve $y = f(x)$. [3]

(ii) In the case where $k = -9$, find the set of values of x for which $f(x) < g(x)$. [3]

(iii) In the case where $k = -1$, find $g^{-1}f(x)$ and solve the equation $g^{-1}f(x) = 0$. [3]

(iv) Express $f(x)$ in the form $2(x + a)^2 + b$, where a and b are constants, and hence state the least value of $f(x)$. [3]

M/J/2006/Q11

Functions f and g are defined by

$$\mathrm{f} : x \mapsto k - x \quad \text{for } x \in \mathbb{R}, \text{ where } k \text{ is a constant,}$$

$$\mathrm{g} : x \mapsto \frac{9}{x+2} \quad \text{for } x \in \mathbb{R},\ x \neq -2.$$

(i) Find the values of k for which the equation $\mathrm{f}(x) = \mathrm{g}(x)$ has two equal roots and solve the equation $\mathrm{f}(x) = \mathrm{g}(x)$ in these cases. [6]

(ii) Solve the equation $\mathrm{fg}(x) = 5$ when $k = 6$. [3]

(iii) Express $\mathrm{g}^{-1}(x)$ in terms of x. [2]

O/N/2006/Q10

The function f is defined by $f : x \mapsto x^2 - 3x$ for $x \in \mathbb{R}$.

(i) Find the set of values of x for which $f(x) > 4$. [3]

(ii) Express $f(x)$ in the form $(x - a)^2 - b$, stating the values of a and b. [2]

(iii) Write down the range of f. [1]

(iv) State, with a reason, whether f has an inverse. [1]

The function g is defined by $g : x \mapsto x - 3\sqrt{x}$ for $x \geqslant 0$.

(v) Solve the equation $g(x) = 10$. [3]

M/J/2008/Q8

Functions f and g are defined by

$$\mathrm{f} : x \mapsto 4x - 2k \quad \text{for } x \in \mathbb{R}, \text{ where } k \text{ is a constant,}$$

$$\mathrm{g} : x \mapsto \frac{9}{2 - x} \quad \text{for } x \in \mathbb{R},\ x \neq 2.$$

(i) Find the values of k for which the equation $\mathrm{fg}(x) = x$ has two equal roots. [4]

(ii) Determine the roots of the equation $\mathrm{fg}(x) = x$ for the values of k found in part **(i)**. [3]

M.JAWAD AMIN

M/J/2010/Q3

The functions f and g are defined for $x \in \mathbb{R}$ by

$$f : x \mapsto 4x - 2x^2,$$
$$g : x \mapsto 5x + 3.$$

(i) Find the range of f. [2]

(ii) Find the value of the constant k for which the equation $gf(x) = k$ has equal roots. [3]

M/J/2014/Q10

Functions f and g are defined by

$$f : x \mapsto 2x - 3, \quad x \in \mathbb{R},$$
$$g : x \mapsto x^2 + 4x, \quad x \in \mathbb{R}.$$

(i) Solve the equation $ff(x) = 11$. [2]

(ii) Find the range of g. [2]

(iii) Find the set of values of x for which $g(x) > 12$. [3]

(iv) Find the value of the constant p for which the equation $gf(x) = p$ has two equal roots. [3]

Function h is defined by $h : x \mapsto x^2 + 4x$ for $x \geqslant k$, and it is given that h has an inverse.

(v) State the smallest possible value of k. [1]

(vi) Find an expression for $h^{-1}(x)$. [4]

M.JAWAD AMIN

O/N/2017/Q2

A function f is defined by $f : x \mapsto 4 - 5x$ for $x \in \mathbb{R}$.

(i) Find an expression for $f^{-1}(x)$ and find the point of intersection of the graphs of $y = f(x)$ and $y = f^{-1}(x)$. [3]

(ii) Sketch, on the same diagram, the graphs of $y = f(x)$ and $y = f^{-1}(x)$, making clear the relationship between the graphs. [3]

M.JAWAD AMIN

M/J/2012/Q10

Functions f and g are defined by

$$\mathrm{f} : x \mapsto 2x + 5 \quad \text{for } x \in \mathbb{R},$$

$$\mathrm{g} : x \mapsto \frac{8}{x - 3} \quad \text{for } x \in \mathbb{R},\ x \neq 3.$$

(i) Obtain expressions, in terms of x, for $\mathrm{f}^{-1}(x)$ and $\mathrm{g}^{-1}(x)$, stating the value of x for which $\mathrm{g}^{-1}(x)$ is not defined. [4]

(ii) Sketch the graphs of $y = \mathrm{f}(x)$ and $y = \mathrm{f}^{-1}(x)$ on the same diagram, making clear the relationship between the two graphs. [3]

(iii) Given that the equation $\mathrm{fg}(x) = 5 - kx$, where k is a constant, has no solutions, find the set of possible values of k. [5]

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O/N/2008/Q10

The function f is defined by

$$f : x \mapsto 3x - 2 \quad \text{for } x \in \mathbb{R}.$$

(i) Sketch, in a single diagram, the graphs of $y = f(x)$ and $y = f^{-1}(x)$, making clear the relationship between the two graphs. [2]

The function g is defined by

$$g : x \mapsto 6x - x^2 \quad \text{for } x \in \mathbb{R}.$$

(ii) Express $gf(x)$ in terms of x, and hence show that the maximum value of $gf(x)$ is 9. [5]

The function h is defined by

$$h : x \mapsto 6x - x^2 \quad \text{for } x \geqslant 3.$$

(iii) Express $6x - x^2$ in the form $a - (x - b)^2$, where a and b are positive constants. [2]

(iv) Express $h^{-1}(x)$ in terms of x. [3]