Discrete Random Variable

O/N/2006/Q2

The discrete random variable X has the following probability distribution.

x	0	1	2	3	4
P(X = x)	0.26	q	3q	0.05	0.09

(i) Find the value of q. [2]

(ii) Find E(X) and Var(X). [3]

O/N/2007/Q2

The random variable X takes the values -2, 0 and 4 only. It is given that P(X = -2) = 2p, P(X = 0) = p and P(X = 4) = 3p.

(i) Find p. [2]

(ii) Find E(X) and Var(X). [4]

M/J/2009/Q2

Gohan throws a fair tetrahedral die with faces numbered 1, 2, 3, 4. If she throws an even number then her score is the number thrown. If she throws an odd number then she throws again and her score is the sum of both numbers thrown. Let the random variable X denote Gohan's score.

(i) Show that
$$P(X = 2) = \frac{5}{16}$$
. [2]

(ii) The table below shows the probability distribution of X.

x	2	3	4	5	6	7
P(X = x)	<u>5</u> 16	$\frac{1}{16}$	3 8	1/8	$\frac{1}{16}$	$\frac{1}{16}$

Calculate E(X) and Var(X). [4]

O/N/2009/Q2

The probability distribution of the random variable X is shown in the following table.

x	-2	-1	0	1	2	3
P(X = x)	0.08	p	0.12	0.16	q	0.22

The mean of X is 1.05.

(i) Write down two equations involving p and q and hence find the values of p and q. [4]

(ii) Find the variance of X. [2]

O/N/2010/Q1

The discrete random variable X takes the values 1, 4, 5, 7 and 9 only. The probability distribution of X is shown in the table.

х	1	4	5	7	9
P(X = x)	4p	$5p^{2}$	1.5 <i>p</i>	2.5p	1.5 <i>p</i>

Find p. [3]

M/J/2012/Q2

The random variable X has the probability distribution shown in the table.

х	2	4	6
P(X = x)	0.5	0.4	0.1

Two independent values of X are chosen at random. The random variable Y takes the value 0 if the two values of X are the same. Otherwise the value of Y is the larger value of X minus the smaller value of X.

(i) Draw up the probability distribution table for Y. [4]

(ii) Find the expected value of Y. [1]

M/J/2017/Q3

In a probability distribution the random variable X takes the value x with probability kx^2 , where k is a constant and x takes values -2, -1, 2, 4 only.

- (i) Show that P(X = -2) has the same value as P(X = 2). [1]
- (ii) Draw up the probability distribution table for X, in terms of k, and find the value of k. [3]
- (iii) Find E(X). [2]

O/N/2017/Q3

A box contains 6 identical-sized discs, of which 4 are blue and 2 are red. Discs are taken at random from the box in turn and not replaced. Let *X* be the number of discs taken, up to and including the first blue one.

(i) Show that
$$P(X = 3) = \frac{1}{15}$$
. [2]

(ii) Draw up the probability distribution table for X. [3]

M/J/2008/Q6

Every day Eduardo tries to phone his friend. Every time he phones there is a 50% chance that his friend will answer. If his friend answers, Eduardo does not phone again on that day. If his friend does not answer, Eduardo tries again in a few minutes' time. If his friend has not answered after 4 attempts, Eduardo does not try again on that day.

- (i) Draw a tree diagram to illustrate this situation.
- (ii) Let X be the number of unanswered phone calls made by Eduardo on a day. Copy and complete the table showing the probability distribution of X.[4]

х	0	1	2	3	4
P(X = x)		$\frac{1}{4}$			

(iii) Calculate the expected number of unanswered phone calls on a day. [2]

M/J/2013/Q7

Susan has a bag of sweets containing 7 chocolates and 5 toffees. Ahmad has a bag of sweets containing 3 chocolates, 4 toffees and 2 boiled sweets. A sweet is taken at random from Susan's bag and put in Ahmad's bag. A sweet is then taken at random from Ahmad's bag.

- (i) Find the probability that the two sweets taken are a toffee from Susan's bag and a boiled sweet from Ahmad's bag. [2]
- (ii) Given that the sweet taken from Ahmad's bag is a chocolate, find the probability that the sweet taken from Susan's bag was also a chocolate. [4]
- (iii) The random variable X is the number of times a chocolate is taken. State the possible values of X and draw up a table to show the probability distribution of X. [5]

[3]

M/J/2014/Q4

Coin A is weighted so that the probability of throwing a head is $\frac{2}{3}$. Coin B is weighted so that the probability of throwing a head is $\frac{1}{4}$. Coin A is thrown twice and coin B is thrown once.

- (i) Show that the probability of obtaining exactly 1 head and 2 tails is $\frac{13}{36}$. [3]
- (ii) Draw up the probability distribution table for the number of heads obtained. [4]
- (iii) Find the expectation of the number of heads obtained. [2]

O/N/2014/Q4

Sharik attempts a multiple choice revision question on-line. There are 3 suggested answers, one of which is correct. When Sharik chooses an answer the computer indicates whether the answer is right or wrong. Sharik first chooses one of the three suggested answers at random. If this answer is wrong he has a second try, choosing an answer at random from the remaining 2. If this answer is also wrong Sharik then chooses the remaining answer, which must be correct.

- (i) Draw a fully labelled tree diagram to illustrate the various choices that Sharik can make until the computer indicates that he has answered the question correctly. [4]
- (ii) The random variable X is the number of attempts that Sharik makes up to and including the one that the computer indicates is correct. Draw up the probability distribution table for X and find E(X).

M/J/2019/Q5

Maryam has 7 sweets in a tin; 6 are toffees and 1 is a chocolate. She chooses one sweet at random and takes it out. Her friend adds 3 chocolates to the tin. Then Maryam takes another sweet at random out of the tin.

- (i) Draw a fully labelled tree diagram to illustrate this situation. [3]
- (ii) Draw up the probability distribution table for the number of toffees taken. [3]
- (iii) Find the mean number of toffees taken. [1]
- (iv) Find the probability that the first sweet taken is a chocolate, given that the second sweet taken is a toffee. [4]

O/N/2018/Q6

A fair red spinner has 4 sides, numbered 1, 2, 3, 4. A fair blue spinner has 3 sides, numbered 1, 2, 3. When a spinner is spun, the score is the number on the side on which it lands. The spinners are spun at the same time. The random variable X denotes the score on the red spinner minus the score on the blue spinner.

(i) Draw up the probability distribution table for X. [3]

(ii) Find Var(X). [3]

(iii) Find the probability that X is equal to 1, given that X is non-zero. [3]

O/N/2019/Q5

A fair red spinner has four sides, numbered 1, 2, 3, 3. A fair blue spinner has three sides, numbered -1, 0, 2. When a spinner is spun, the score is the number on the side on which it lands. The spinners are spun at the same time. The random variable X denotes the score on the red spinner minus the score on the blue spinner.

(i) Draw up the probability distribution table for X. [4]

(ii) Find Var(X). [3]

M/J/2011/Q7

Judy and Steve play a game using five cards numbered 3, 4, 5, 8, 9. Judy chooses a card at random, looks at the number on it and replaces the card. Then Steve chooses a card at random, looks at the number on it and replaces the card. If their two numbers are equal the score is 0. Otherwise, the smaller number is subtracted from the larger number to give the score.

- (i) Show that the probability that the score is 6 is 0.08.
- (ii) Draw up a probability distribution table for the score. [2]
- (iii) Calculate the mean score. [1]

If the score is 0 they play again. If the score is 4 or more Judy wins. Otherwise Steve wins. They continue playing until one of the players wins.

- (iv) Find the probability that Judy wins with the second choice of cards. [3]
- (v) Find an expression for the probability that Judy wins with the *n*th choice of cards. [2]

O/N/2015/Q6

A fair spinner A has edges numbered 1, 2, 3, 3. A fair spinner B has edges numbered -3, -2, -1, 1. Each spinner is spun. The number on the edge that the spinner comes to rest on is noted. Let X be the sum of the numbers for the two spinners.

(i) Copy and complete the table showing the possible values of X.

[1]

		Spinner A						
		1	2	3	3			
Spinner B	-3	-2						
	-2			1				
	-1							
	1							

- (ii) Draw up a table showing the probability distribution of X.
- (iii) Find Var(X). [3]
- (iv) Find the probability that *X* is even, given that *X* is positive.

[3]

M/J/2007/Q7

A vegetable basket contains 12 peppers, of which 3 are red, 4 are green and 5 are yellow. Three peppers are taken, at random and without replacement, from the basket.

- (i) Find the probability that the three peppers are all different colours. [3]
- (ii) Show that the probability that exactly 2 of the peppers taken are green is $\frac{12}{55}$. [2]
- (iii) The number of green peppers taken is denoted by the discrete random variable X. Draw up a probability distribution table for X. [5]

M/J/2010/Q6

A small farm has 5 ducks and 2 geese. Four of these birds are to be chosen at random. The random variable *X* represents the number of geese chosen.

(i) Draw up the probability distribution of
$$X$$
. [3]

(ii) Show that
$$E(X) = \frac{8}{7}$$
 and calculate $Var(X)$. [3]

(iii) When the farmer's dog is let loose, it chases either the ducks with probability ³/₅ or the geese with probability ²/₅. If the dog chases the ducks there is a probability of ¹/₁₀ that they will attack the dog. If the dog chases the geese there is a probability of ³/₄ that they will attack the dog. Given that the dog is not attacked, find the probability that it was chasing the geese. [4]

M/J/2015/Q5

A box contains 5 discs, numbered 1, 2, 4, 6, 7. William takes 3 discs at random, without replacement, and notes the numbers on the discs.

(i) Find the probability that the numbers on the 3 discs are two even numbers and one odd number.

[3]

The smallest of the numbers on the 3 discs taken is denoted by the random variable S.

(ii) By listing all possible selections (126, 246 and so on) draw up the probability distribution table for *S*. [5]

M/J/2018/Q4

Mrs Rupal chooses 3 animals at random from 5 dogs and 2 cats. The random variable *X* is the number of cats chosen.

(i) Draw up the probability distribution table for X.

[4]

(ii) You are given that $E(X) = \frac{6}{7}$. Find the value of Var(X). [2]

O/N/2012/Q6

A fair tetrahedral die has four triangular faces, numbered 1, 2, 3 and 4. The score when this die is thrown is the number on the face that the die lands on. This die is thrown three times. The random variable X is the sum of the three scores.

(i) Show that
$$P(X = 9) = \frac{10}{64}$$
. [3]

(ii) Copy and complete the probability distribution table for X.

x	3	4	5	6	7	8	9	10	11	12
P(X = x)	1 64	3 64			12 64					

(iii) Event *R* is 'the sum of the three scores is 9'. Event *S* is 'the product of the three scores is 16'. Determine whether events *R* and *S* are independent, showing your working. [5]

O/N/2013/Q7

Rory has 10 cards. Four of the cards have a 3 printed on them and six of the cards have a 4 printed on them. He takes three cards at random, without replacement, and adds up the numbers on the cards.

(i) Show that P(the sum of the numbers on the three cards is 11) =
$$\frac{1}{2}$$
. [3]

(ii) Draw up a probability distribution table for the sum of the numbers on the three cards. [4]

Event *R* is 'the sum of the numbers on the three cards is 11'. Event *S* is 'the number on the first card taken is a 3'.

(iii) Determine whether events
$$R$$
 and S are independent. Justify your answer. [3]

(iv) Determine whether events
$$R$$
 and S are exclusive. Justify your answer. [1]

[3]