

NORMAL DISTRIBUTION

O/N//2012/Q2

The random variable X is the daily profit, in thousands of dollars, made by a company. X is normally distributed with mean 6.4 and standard deviation 5.2.

- (i) Find the probability that, on a randomly chosen day, the company makes a profit between \$10 000 and \$12 000. [3]

O/N/2013/Q1

It is given that $X \sim N(1.5, 3.2^2)$. Find the probability that a randomly chosen value of X is less than -2.4 . [3]

M/J/2010/Q2

The lengths of new pencils are normally distributed with mean 11 cm and standard deviation 0.095 cm.

Find the probability that a pencil chosen at random has a length greater than 10.9 cm. [2]

M/J/2005/Q1

It is known that, on average, 2 people in 5 in a certain country are overweight. A random sample of 400 people is chosen. Using a suitable approximation, find the probability that fewer than 165 people in the sample are overweight. [5]

O/N/2005/Q7

In tests on a new type of light bulb it was found that the time they lasted followed a normal distribution with standard deviation 40.6 hours. 10% lasted longer than 5130 hours.

- (i) Find the mean lifetime, giving your answer to the nearest hour. [3]
- (ii) Find the probability that a light bulb fails to last for 5000 hours. [3]
- (iii) A hospital buys 600 of these light bulbs. Using a suitable approximation, find the probability that fewer than 65 light bulbs will last longer than 5130 hours. [4]

M/J/2006/Q7

A survey of adults in a certain large town found that 76% of people wore a watch on their left wrist, 15% wore a watch on their right wrist and 9% did not wear a watch.

- (i) A random sample of 14 adults was taken. Find the probability that more than 2 adults did not wear a watch. [4]
- (ii) A random sample of 200 adults was taken. Using a suitable approximation, find the probability that more than 155 wore a watch on their left wrist. [5]

O/N/2006/Q7

A manufacturer makes two sizes of elastic bands: large and small. 40% of the bands produced are large bands and 60% are small bands. Assuming that each pack of these elastic bands contains a random selection, calculate the probability that, in a pack containing 20 bands, there are

- (i) equal numbers of large and small bands, [2]
- (ii) more than 17 small bands. [3]

An office pack contains 150 elastic bands.

- (iii) Using a suitable approximation, calculate the probability that the number of small bands in the office pack is between 88 and 97 inclusive. [6]

M/J/2007/Q6

The probability that New Year's Day is on a Saturday in a randomly chosen year is $\frac{1}{7}$.

- (i) 15 years are chosen randomly. Find the probability that at least 3 of these years have New Year's Day on a Saturday. [4]
- (ii) 56 years are chosen randomly. Use a suitable approximation to find the probability that more than 7 of these years have New Year's Day on a Saturday. [5]

M/J/2008/Q7

A die is biased so that the probability of throwing a 5 is 0.75 and the probabilities of throwing a 1, 2, 3, 4 or 6 are all equal.

- (i) The die is thrown three times. Find the probability that the result is a 1 followed by a 5 followed by any even number. [3]
- (ii) Find the probability that, out of 10 throws of this die, at least 8 throws result in a 5. [3]
- (iii) The die is thrown 90 times. Using an appropriate approximation, find the probability that a 5 is thrown more than 60 times. [5]

O/N/2008/Q2

On a production line making toys, the probability of any toy being faulty is 0.08. A random sample of 200 toys is checked. Use a suitable approximation to find the probability that there are at least 15 faulty toys. [5]

M/J/2009/Q3

On a certain road 20% of the vehicles are trucks, 16% are buses and the remainder are cars.

- (i) A random sample of 11 vehicles is taken. Find the probability that fewer than 3 are buses. [3]
- (ii) A random sample of 125 vehicles is now taken. Using a suitable approximation, find the probability that more than 73 are cars. [5]

O/N/2010/Q6

- (i) State three conditions that must be satisfied for a situation to be modelled by a binomial distribution. [2]

On any day, there is a probability of 0.3 that Julie's train is late.

- (ii) Nine days are chosen at random. Find the probability that Julie's train is late on more than 7 days or fewer than 2 days. [3]
- (iii) 90 days are chosen at random. Find the probability that Julie's train is late on more than 35 days or fewer than 27 days. [5]

M/J/2011/Q2

In Scotland, in November, on average 80% of days are cloudy. Assume that the weather on any one day is independent of the weather on other days.

- (i) Use a normal approximation to find the probability of there being fewer than 25 cloudy days in Scotland in November (30 days). [4]
- (ii) Give a reason why the use of a normal approximation is justified. [1]

O/N/2013/Q5

On trains in the morning rush hour, each person is either a student with probability 0.36, or an office worker with probability 0.22, or a shop assistant with probability 0.29 or none of these.

- (i) 8 people on a morning rush hour train are chosen at random. Find the probability that between 4 and 6 inclusive are office workers. [3]
- (ii) 300 people on a morning rush hour train are chosen at random. Find the probability that between 31 and 49 inclusive are neither students nor office workers nor shop assistants. [6]

M/J/2014/Q7

The time Rafa spends on his homework each day in term-time has a normal distribution with mean 1.9 hours and standard deviation σ hours. On 80% of these days he spends more than 1.35 hours on his homework.

- (i) Find the value of σ . [3]
- (ii) Find the probability that, on a randomly chosen day in term-time, Rafa spends less than 2 hours on his homework. [2]
- (iii) A random sample of 200 days in term-time is taken. Use an approximation to find the probability that the number of days on which Rafa spends more than 1.35 hours on his homework is between 163 and 173 inclusive. [6]

O/N/2017/Q5

Blank CDs are packed in boxes of 30. The probability that a blank CD is faulty is 0.04. A box is rejected if more than 2 of the blank CDs are faulty.

- (i) Find the probability that a box is rejected. [3]
- (ii) 280 boxes are chosen randomly. Use an approximation to find the probability that at least 30 of these boxes are rejected. [5]

M/J/2018/Q7

In a certain country, 60% of mobile phones sold are made by Company *A*, 35% are made by Company *B* and 5% are made by other companies.

- (i) Find the probability that, out of a random sample of 13 people who buy a mobile phone, fewer than 11 choose a mobile phone made by Company *A*. [3]

- (ii) Use a suitable approximation to find the probability that, out of a random sample of 130 people who buy a mobile phone, at least 50 choose a mobile phone made by Company *B*. [5]

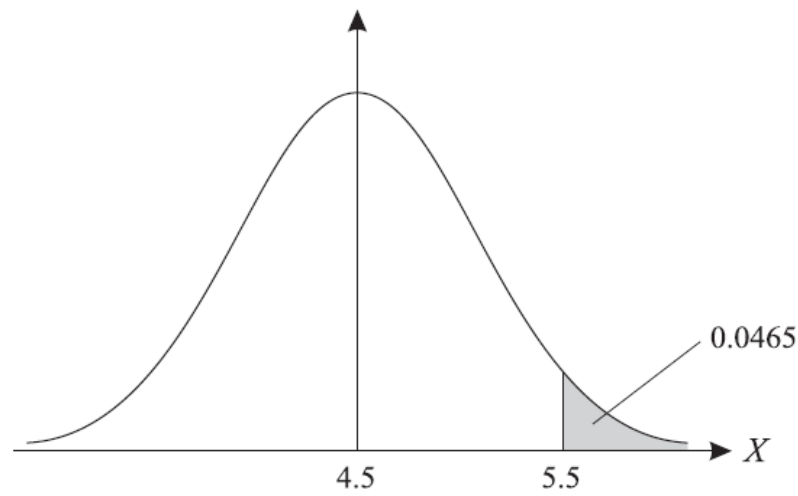
- (iii) A random sample of n mobile phones sold is chosen. The probability that at least one of these phones is made by Company *B* is more than 0.98. Find the least possible value of n . [3]

O/N/2019/Q4

In Quarendon, 66% of households are satisfied with the speed of their wifi connection.

- (i) Find the probability that, out of 10 households chosen at random in Quarendon, at least 8 are satisfied with the speed of their wifi connection. [3]

- (ii) A random sample of 150 households in Quarendon is chosen. Use a suitable approximation to find the probability that more than 84 are satisfied with the speed of their wifi connection. [5]



The random variable X has a normal distribution with mean 4.5. It is given that $P(X > 5.5) = 0.0465$ (see diagram).

- (i) Find the standard deviation of X . [3]
- (ii) Find the probability that a random observation of X lies between 3.8 and 4.8. [4]

O/N/2006/Q5

- (i) Give an example of a variable in real life which could be modelled by a normal distribution. [1]
- (ii) The random variable X is normally distributed with mean μ and variance 21.0. Given that $P(X > 10.0) = 0.7389$, find the value of μ . [3]
- (iii) If 300 observations are taken at random from the distribution in part (ii), estimate how many of these would be greater than 22.0. [4]

M/J/2008/Q4

In a certain country the time taken for a common infection to clear up is normally distributed with mean μ days and standard deviation 2.6 days. 25% of these infections clear up in less than 7 days.

(i) Find the value of μ . [4]

In another country the standard deviation of the time taken for the infection to clear up is the same as in part (i), but the mean is 6.5 days. The time taken is normally distributed.

(ii) Find the probability that, in a randomly chosen case from this country, the infection takes longer than 6.2 days to clear up. [3]

O/N/2008/Q3

(i) The daily minimum temperature in degrees Celsius ($^{\circ}\text{C}$) in January in Ottawa is a random variable with distribution $N(-15.1, 62.0)$. Find the probability that a randomly chosen day in January in Ottawa has a minimum temperature above 0°C . [3]

(ii) In another city the daily minimum temperature in $^{\circ}\text{C}$ in January is a random variable with distribution $N(\mu, 40.0)$. In this city the probability that a randomly chosen day in January has a minimum temperature above 0°C is 0.8888. Find the value of μ . [3]

M/J/2009/Q1

The volume of milk in millilitres in cartons is normally distributed with mean μ and standard deviation 8. Measurements were taken of the volume in 900 of these cartons and it was found that 225 of them contained more than 1002 millilitres.

- (i) Calculate the value of μ . [3]
- (ii) Three of these 900 cartons are chosen at random. Calculate the probability that exactly 2 of them contain more than 1002 millilitres. [2]

M/J/2010/Q4

The random variable X is normally distributed with mean μ and standard deviation σ .

- (i) Given that $5\sigma = 3\mu$, find $P(X < 2\mu)$. [3]
- (ii) With a different relationship between μ and σ , it is given that $P(X < \frac{1}{3}\mu) = 0.8524$. Express μ in terms of σ . [3]

O/N/2010/Q5

The distance the Zotoc car can travel on 20 litres of fuel is normally distributed with mean 320 km and standard deviation 21.6 km. The distance the Ganmor car can travel on 20 litres of fuel is normally distributed with mean 350 km and standard deviation 7.5 km. Both cars are filled with 20 litres of fuel and are driven towards a place 367 km away.

- (i) For each car, find the probability that it runs out of fuel before it has travelled 367 km. [3]
- (ii) The probability that a Zotoc car can travel at least $(320 + d)$ km on 20 litres of fuel is 0.409. Find the value of d . [4]

M/J/2011/Q6

The lengths, in centimetres, of drinking straws produced in a factory have a normal distribution with mean μ and variance 0.64. It is given that 10% of the straws are shorter than 20 cm.

- (i) Find the value of μ . [3]
- (ii) Find the probability that, of 4 straws chosen at random, fewer than 2 will have a length between 21.5 cm and 22.5 cm. [6]

O/N/2011/Q7

- (i) In a certain country, the daily minimum temperature, in $^{\circ}\text{C}$, in winter has the distribution $N(8, 24)$. Find the probability that a randomly chosen winter day in this country has a minimum temperature between 7°C and 12°C . [3]

The daily minimum temperature, in $^{\circ}\text{C}$, in another country in winter has a normal distribution with mean μ and standard deviation 2μ .

- (ii) Find the proportion of winter days on which the minimum temperature is below zero. [2]
- (iii) 70 winter days are chosen at random. Find how many of these would be expected to have a minimum temperature which is more than three times the mean. [3]
- (iv) The probability of the minimum temperature being above 6°C on any winter day is 0.0735. Find the value of μ . [3]

O/N/2012/Q4

The mean of a certain normally distributed variable is four times the standard deviation. The probability that a randomly chosen value is greater than 5 is 0.15.

- (i) Find the mean and standard deviation. [4]
- (ii) 200 values of the variable are chosen at random. Find the probability that at least 160 of these values are less than 5. [5]

M/J/2013/Q1

The random variable Y is normally distributed with mean equal to five times the standard deviation. It is given that $P(Y > 20) = 0.0732$. Find the mean. [3]

O/N/2013/Q3

The amount of fibre in a packet of a certain brand of cereal is normally distributed with mean 160 grams. 19% of packets of cereal contain more than 190 grams of fibre.

- (i) Find the standard deviation of the amount of fibre in a packet. [3]
- (ii) Kate buys 12 packets of cereal. Find the probability that at least 1 of the packets contains more than 190 grams of fibre. [2]

O/N/2014/Q5

- (a) The time, X hours, for which people sleep in one night has a normal distribution with mean 7.15 hours and standard deviation 0.88 hours.
- (i) Find the probability that a randomly chosen person sleeps for less than 8 hours in a night. [2]
- (ii) Find the value of q such that $P(X < q) = 0.75$. [3]
- (b) The random variable Y has the distribution $N(\mu, \sigma^2)$, where $2\sigma = 3\mu$ and $\mu \neq 0$. Find $P(Y > 4\mu)$. [3]

M/J/2015/Q7

- (a) Once a week Zak goes for a run. The time he takes, in minutes, has a normal distribution with mean 35.2 and standard deviation 4.7.
- (i) Find the expected number of days during a year (52 weeks) for which Zak takes less than 30 minutes for his run. [4]
- (ii) The probability that Zak's time is between 35.2 minutes and t minutes, where $t > 35.2$, is 0.148. Find the value of t . [3]
- (b) The random variable X has the distribution $N(\mu, \sigma^2)$. It is given that $P(X < 7) = 0.2119$ and $P(X < 10) = 0.6700$. Find the values of μ and σ . [5]

O/N/2015/Q7

- (a) A petrol station finds that its daily sales, in litres, are normally distributed with mean 4520 and standard deviation 560.
- (i) Find on how many days of the year (365 days) the daily sales can be expected to exceed 3900 litres. [4]

The daily sales at another petrol station are X litres, where X is normally distributed with mean m and standard deviation 560. It is given that $P(X > 8000) = 0.122$.

- (ii) Find the value of m . [3]
- (iii) Find the probability that daily sales at this petrol station exceed 8000 litres on fewer than 2 of 6 randomly chosen days. [3]
- (b) The random variable Y is normally distributed with mean μ and standard deviation σ . Given that $\sigma = \frac{2}{3}\mu$, find the probability that a random value of Y is less than 2μ . [3]

M/J/2018/Q3

- (i) The volume of soup in Super Soup cartons has a normal distribution with mean μ millilitres and standard deviation 9 millilitres. Tests have shown that 10% of cartons contain less than 440 millilitres of soup. Find the value of μ . [3]
- (ii) A food retailer orders 150 Super Soup cartons. Calculate the number of these cartons for which you would expect the volume of soup to be more than 1.8 standard deviations above the mean. [3]

M/J/2019/Q4

It is known that 20% of male giant pandas in a certain area weigh more than 121 kg and 71.9% weigh more than 102 kg. Weights of male giant pandas in this area have a normal distribution. Find the mean and standard deviation of the weights of male giant pandas in this area. [5]

O/N/2019/Q6

The heights, in metres, of fir trees in a large forest have a normal distribution with mean 40 and standard deviation 8.

- (i) Find the probability that a fir tree chosen at random in this forest has a height less than 45 metres. [2]
- (ii) Find the probability that a fir tree chosen at random in this forest has a height within 5 metres of the mean. [2]

In another forest, the heights of another type of fir tree are modelled by a normal distribution. A scientist measures the heights of 500 randomly chosen trees of this type. He finds that 48 trees are less than 10 m high and 76 trees are more than 24 m high.

- (iii) Find the mean and standard deviation of the heights of trees of this type. [5]

O/N/2018/Q7

- (a) The time, X hours, for which students use a games machine in any given day has a normal distribution with mean 3.24 hours and standard deviation 0.96 hours.
- (i) On how many days of the year (365 days) would you expect a randomly chosen student to use a games machine for less than 4 hours? [3]
- (ii) Find the value of k such that $P(X > k) = 0.2$. [3]
- (iii) Find the probability that the number of hours for which a randomly chosen student uses a games machine in a day is within 1.5 standard deviations of the mean. [3]
- (b) The variable Y is normally distributed with mean μ and standard deviation σ , where $4\sigma = 3\mu$ and $\mu \neq 0$. Find the probability that a randomly chosen value of Y is positive. [3]

O/N/2017/Q7

In Jimpuri the weights, in kilograms, of boys aged 16 years have a normal distribution with mean 61.4 and standard deviation 12.3.

- (i) Find the probability that a randomly chosen boy aged 16 years in Jimpuri weighs more than 65 kilograms. [3]
- (ii) For boys aged 16 years in Jimpuri, 25% have a weight between 65 kilograms and k kilograms, where k is greater than 65. Find k . [4]

In Brigville the weights, in kilograms, of boys aged 16 years have a normal distribution. 99% of the boys weigh less than 97.2 kilograms and 33% of the boys weigh less than 55.2 kilograms.

- (iii) Find the mean and standard deviation of the weights of boys aged 16 years in Brigville. [5]

M/J/2017/Q5

The lengths of videos of a certain popular song have a normal distribution with mean 3.9 minutes. 18% of these videos last for longer than 4.2 minutes.

- (i) Find the standard deviation of the lengths of these videos. [3]
- (ii) Find the probability that the length of a randomly chosen video differs from the mean by less than half a minute. [4]

The lengths of videos of another popular song have a normal distribution with the same mean of 3.9 minutes but the standard deviation is twice the standard deviation in part (i). The probability that the length of a randomly chosen video of this song differs from the mean by less than half a minute is denoted by p .

- (iii) Without any further calculation, determine whether p is more than, equal to, or less than your answer to part (ii). You must explain your reasoning. [2]

M/J/2012/Q7

The times taken to play Beethoven's Sixth Symphony can be assumed to have a normal distribution with mean 41.1 minutes and standard deviation 3.4 minutes. Three occasions on which this symphony is played are chosen at random.

- (i) Find the probability that the symphony takes longer than 42 minutes to play on exactly 1 of these occasions. [4]

The times taken to play Beethoven's Fifth Symphony can also be assumed to have a normal distribution. The probability that the time is less than 26.5 minutes is 0.1, and the probability that the time is more than 34.6 minutes is 0.05.

- (ii) Find the mean and standard deviation of the times to play this symphony. [5]
- (iii) Assuming that the times to play the two symphonies are independent of each other, find the probability that, when both symphonies are played, both of the times are less than 34.6 minutes. [4]

M/J/2007/Q2

- (a) The random variable X is normally distributed. The mean is twice the standard deviation. It is given that $P(X > 5.2) = 0.9$. Find the standard deviation. [4]
- (b) A normal distribution has mean μ and standard deviation σ . If 800 observations are taken from this distribution, how many would you expect to be between $\mu - \sigma$ and $\mu + \sigma$? [3]

M/J/2005/Q6

Tyre pressures on a certain type of car independently follow a normal distribution with mean 1.9 bars and standard deviation 0.15 bars.

- (i) Find the probability that all four tyres on a car of this type have pressures between 1.82 bars and 1.92 bars. [5]
- (ii) Safety regulations state that the pressures must be between $1.9 - b$ bars and $1.9 + b$ bars. It is known that 80% of tyres are within these safety limits. Find the safety limits. [3]

