

"A car of mass 1200 kg travels along a horizontal straight road. The power provided by the car's engine is constant and equal to 20 kW. The resistance to the car's motion is constant and equal to 500 N. The car passes through the points A and B with speeds  $10 \text{ m s}^{-1}$  and  $25 \text{ m s}^{-1}$  respectively. The car takes 30.5 s to travel from A to B.

(i) Find the acceleration of the car at A.

[4]

(ii) By considering work and energy, find the distance AB.

[8]"

A crate of mass 50 kg is dragged along a horizontal floor by a constant force of magnitude 400 N acting at an angle  $\alpha^\circ$  upwards from the horizontal. The total resistance to motion of the crate has constant magnitude 250 N. The crate starts from rest at the point  $O$  and passes the point  $P$  with a speed of  $2 \text{ m s}^{-1}$ . The distance  $OP$  is 20 m. For the crate's motion from  $O$  to  $P$ , find

- (i) the increase in kinetic energy of the crate, [1]
- (ii) the work done against the resistance to the motion of the crate, [1]
- (iii) the value of  $\alpha$ . [3]

A block of mass 50 kg is pulled up a straight hill and passes through points  $A$  and  $B$  with speeds  $7 \text{ m s}^{-1}$  and  $3 \text{ m s}^{-1}$  respectively. The distance  $AB$  is 200 m and  $B$  is 15 m higher than  $A$ . For the motion of the block from  $A$  to  $B$ , find

(i) the loss in kinetic energy of the block, [2]

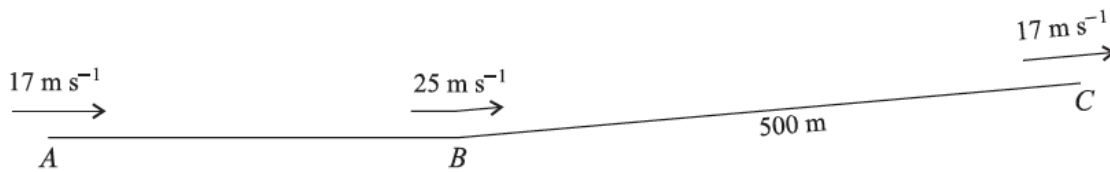
(ii) the gain in potential energy of the block. [2]

The resistance to motion of the block has magnitude 7.5 N.

(iii) Find the work done by the pulling force acting on the block. [2]

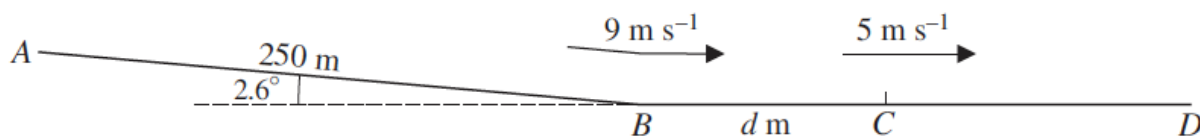
The pulling force acting on the block has constant magnitude 45 N and acts at an angle  $\alpha^\circ$  upwards from the hill.

(iv) Find the value of  $\alpha$ . [3]



A lorry of mass  $12\,500\text{ kg}$  travels along a road that has a straight horizontal section  $AB$  and a straight inclined section  $BC$ . The length of  $BC$  is  $500\text{ m}$ . The speeds of the lorry at  $A$ ,  $B$  and  $C$  are  $17\text{ m s}^{-1}$ ,  $25\text{ m s}^{-1}$  and  $17\text{ m s}^{-1}$  respectively (see diagram).

- (i) The work done against the resistance to motion of the lorry, as it travels from  $A$  to  $B$ , is  $5000\text{ kJ}$ . Find the work done by the driving force as the lorry travels from  $A$  to  $B$ . [4]
- (ii) As the lorry travels from  $B$  to  $C$ , the resistance to motion is  $4800\text{ N}$  and the work done by the driving force is  $3300\text{ kJ}$ . Find the height of  $C$  above the level of  $AB$ . [4]



A cyclist and his machine have a total mass of 80 kg. The cyclist starts from rest at the top  $A$  of a straight path  $AB$ , and freewheels (moves without pedalling or braking) down the path to  $B$ . The path  $AB$  is inclined at  $2.6^\circ$  to the horizontal and is of length 250 m (see diagram).

- (i) Given that the cyclist passes through  $B$  with speed  $9 \text{ m s}^{-1}$ , find the gain in kinetic energy and the loss in potential energy of the cyclist and his machine. Hence find the work done against the resistance to motion of the cyclist and his machine. [3]

The cyclist continues to freewheel along a horizontal straight path  $BD$  until he reaches the point  $C$ , where the distance  $BC$  is  $d$  m. His speed at  $C$  is  $5 \text{ m s}^{-1}$ . The resistance to motion is constant, and is the same on  $BD$  as on  $AB$ .

- (ii) Find the value of  $d$ . [3]

The cyclist starts to pedal at  $C$ , generating 425 W of power.

- (iii) Find the acceleration of the cyclist immediately after passing through  $C$ . [3]

A lorry of mass 15 000 kg moves with constant speed  $14 \text{ m s}^{-1}$  from the top to the bottom of a straight hill of length 900 m. The top of the hill is 18 m above the level of the bottom of the hill. The total work done by the resistive forces acting on the lorry, including the braking force, is  $4.8 \times 10^6 \text{ J}$ . Find

(i) the loss in gravitational potential energy of the lorry, [1]

(ii) the work done by the driving force. [1]

On reaching the bottom of the hill the lorry continues along a straight horizontal road against a constant resistance of 1600 N. There is no braking force acting. The speed of the lorry increases from  $14 \text{ m s}^{-1}$  at the bottom of the hill to  $16 \text{ m s}^{-1}$  at the point  $X$ , where  $X$  is 2500 m from the bottom of the hill.

(iii) By considering energy, find the work done by the driving force of the lorry while it travels from the bottom of the hill to  $X$ . [3]

$P$  and  $Q$  are fixed points on a line of greatest slope of an inclined plane. The point  $Q$  is at a height of 0.45 m above the level of  $P$ . A particle of mass 0.3 kg moves upwards along the line  $PQ$ .

- (i) Given that the plane is smooth and that the particle just reaches  $Q$ , find the speed with which it passes through  $P$ . [3]
- (ii) It is given instead that the plane is rough. The particle passes through  $P$  with the same speed as that found in part (i), and just reaches a point  $R$  which is between  $P$  and  $Q$ . The work done against the frictional force in moving from  $P$  to  $R$  is 0.39 J. Find the potential energy gained by the particle in moving from  $P$  to  $R$  and hence find the height of  $R$  above the level of  $P$ . [4]

A block of mass 20 kg is pulled from the bottom to the top of a slope. The slope has length 10 m and is inclined at  $4.5^\circ$  to the horizontal. The speed of the block is  $2.5 \text{ m s}^{-1}$  at the bottom of the slope and  $1.5 \text{ m s}^{-1}$  at the top of the slope.

- (i) Find the loss of kinetic energy and the gain in potential energy of the block. [3]
- (ii) Given that the work done against the resistance to motion is 50 J, find the work done by the pulling force acting on the block. [2]
- (iii) Given also that the pulling force is constant and acts at an angle of  $15^\circ$  upwards from the slope, find its magnitude. [2]



A lorry of mass 16 000 kg climbs a straight hill  $ABCD$  which makes an angle  $\theta$  with the horizontal, where  $\sin \theta = \frac{1}{20}$ . For the motion from  $A$  to  $B$ , the work done by the driving force of the lorry is 1200 kJ and the resistance to motion is constant and equal to 1240 N. The speed of the lorry is  $15 \text{ m s}^{-1}$  at  $A$  and  $12 \text{ m s}^{-1}$  at  $B$ .

(i) Find the distance  $AB$ . [5]

For the motion from  $B$  to  $D$  the gain in potential energy of the lorry is 2400 kJ.

(ii) Find the distance  $BD$ . [1]

For the motion from  $B$  to  $D$  the driving force of the lorry is constant and equal to 7200 N. From  $B$  to  $C$  the resistance to motion is constant and equal to 1240 N and from  $C$  to  $D$  the resistance to motion is constant and equal to 1860 N.

(iii) Given that the speed of the lorry at  $D$  is  $7 \text{ m s}^{-1}$ , find the distance  $BC$ . [4]

A car of mass 1250 kg travels from the bottom to the top of a straight hill which has length 400 m and is inclined to the horizontal at an angle of  $\alpha$ , where  $\sin \alpha = 0.125$ . The resistance to the car's motion is 800 N. Find the work done by the car's engine in each of the following cases.

(i) The car's speed is constant. [4]

(ii) The car's initial speed is  $6 \text{ m s}^{-1}$ , the car's driving force is 3 times greater at the top of the hill than it is at the bottom, and the car's power output is 5 times greater at the top of the hill than it is at the bottom. [5]

$A$  and  $B$  are two points 50 metres apart on a straight path inclined at an angle  $\theta$  to the horizontal, where  $\sin \theta = 0.05$ , with  $A$  above the level of  $B$ . A block of mass 16 kg is pulled down the path from  $A$  to  $B$ . The block starts from rest at  $A$  and reaches  $B$  with a speed of  $10 \text{ m s}^{-1}$ . The work done by the pulling force acting on the block is 1150 J.

- (i) Find the work done against the resistance to motion. [3]

The block is now pulled up the path from  $B$  to  $A$ . The work done by the pulling force and the work done against the resistance to motion are the same as in the case of the downward motion.

- (ii) Show that the speed of the block when it reaches  $A$  is the same as its speed when it started at  $B$ . [2]

A cyclist is riding up a straight hill inclined at an angle  $\alpha$  to the horizontal, where  $\sin \alpha = 0.04$ . The total mass of the bicycle and rider is 80 kg. The cyclist is riding at a constant speed of  $4 \text{ m s}^{-1}$ . There is a force resisting the motion. The work done by the cyclist against this resistance force over a distance of 25 m is 600 J.

(i) Find the power output of the cyclist. [4]

The cyclist reaches the top of the hill, where the road becomes horizontal, with speed  $4 \text{ m s}^{-1}$ . The cyclist continues to work at the same rate on the horizontal part of the road.

(ii) Find the speed of the cyclist 10 seconds after reaching the top of the hill, given that the work done by the cyclist during this period against the resistance force is 1200 J. [4]

A particle of mass  $0.3 \text{ kg}$  is released from rest above a tank containing water. The particle falls vertically, taking  $0.8 \text{ s}$  to reach the water surface. There is no instantaneous change of speed when the particle enters the water. The depth of water in the tank is  $1.25 \text{ m}$ . The water exerts a force on the particle resisting its motion. The work done against this resistance force from the instant that the particle enters the water until it reaches the bottom of the tank is  $1.2 \text{ J}$ .

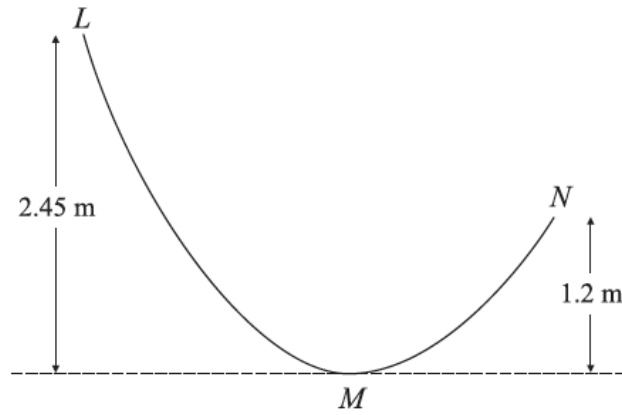
- (i) Use an energy method to find the speed of the particle when it reaches the bottom of the tank.

[4]

When the particle reaches the bottom of the tank, it bounces back vertically upwards with initial speed  $7 \text{ m s}^{-1}$ . As the particle rises through the water, it experiences a constant resistance force of  $1.8 \text{ N}$ . The particle comes to instantaneous rest  $t$  seconds after it bounces on the bottom of the tank.

- (ii) Find the value of  $t$ .

[7]

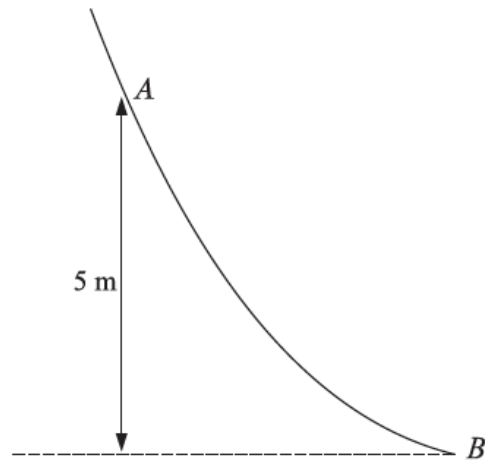


The diagram shows the vertical cross-section  $LMN$  of a fixed smooth surface.  $M$  is the lowest point of the cross-section.  $L$  is 2.45 m above the level of  $M$ , and  $N$  is 1.2 m above the level of  $M$ . A particle of mass 0.5 kg is released from rest at  $L$  and moves on the surface until it leaves it at  $N$ . Find

- (i) the greatest speed of the particle, [3]
- (ii) the kinetic energy of the particle at  $N$ . [2]

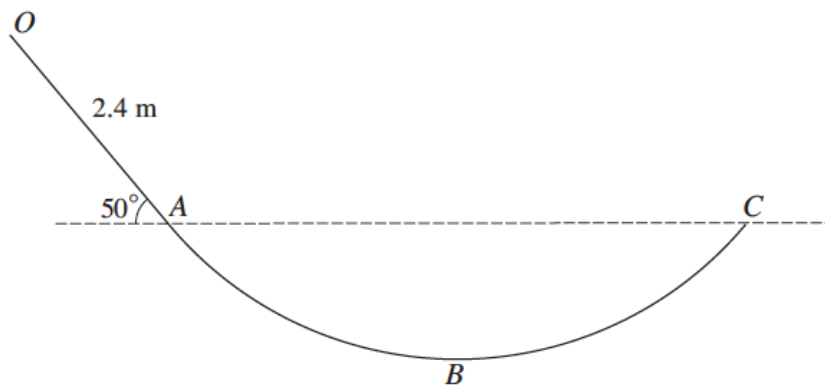
The particle is now projected from  $N$ , with speed  $v \text{ m s}^{-1}$ , along the surface towards  $M$ .

- (iii) Find the least value of  $v$  for which the particle will reach  $L$ . [2]



The diagram shows the vertical cross-section of a surface.  $A$  and  $B$  are two points on the cross-section, and  $A$  is 5 m higher than  $B$ . A particle of mass 0.35 kg passes through  $A$  with speed  $7 \text{ m s}^{-1}$ , moving on the surface towards  $B$ .

- (i) Assuming that there is no resistance to motion, find the speed with which the particle reaches  $B$ . [3]
- (ii) Assuming instead that there is a resistance to motion, and that the particle reaches  $B$  with speed  $11 \text{ m s}^{-1}$ , find the work done against this resistance as the particle moves from  $A$  to  $B$ . [3]



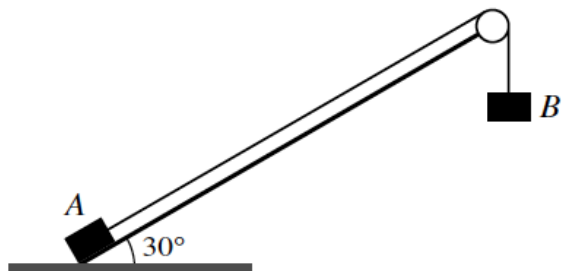
$OABC$  is a vertical cross-section of a smooth surface. The straight part  $OA$  has length  $2.4\text{ m}$  and makes an angle of  $50^\circ$  with the horizontal.  $A$  and  $C$  are at the same horizontal level and  $B$  is the lowest point of the cross-section (see diagram). A particle  $P$  of mass  $0.8\text{ kg}$  is released from rest at  $O$  and moves on the surface.  $P$  remains in contact with the surface until it leaves the surface at  $C$ . Find

- (i) the kinetic energy of  $P$  at  $A$ , [2]  
 (ii) the speed of  $P$  at  $C$ . [2]

The greatest speed of  $P$  is  $8\text{ m s}^{-1}$ .

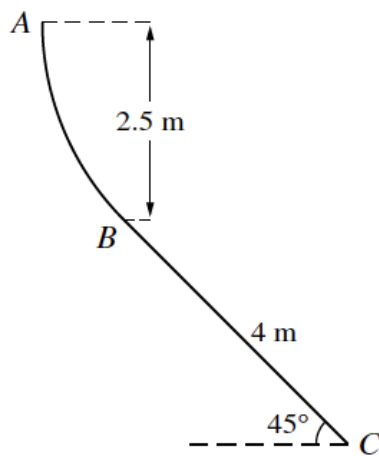
- (iii) Find the depth of  $B$  below the horizontal through  $A$  and  $C$ . [3]



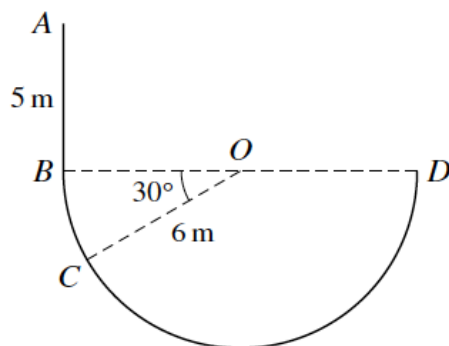


A light inextensible rope has a block  $A$  of mass  $5\text{ kg}$  attached at one end, and a block  $B$  of mass  $16\text{ kg}$  attached at the other end. The rope passes over a smooth pulley which is fixed at the top of a rough plane inclined at an angle of  $30^\circ$  to the horizontal. Block  $A$  is held at rest at the bottom of the plane and block  $B$  hangs below the pulley (see diagram). The coefficient of friction between  $A$  and the plane is  $\frac{1}{\sqrt{3}}$ . Block  $A$  is released from rest and the system starts to move. When each of the blocks has moved a distance of  $x\text{ m}$  each has speed  $v\text{ m s}^{-1}$ .

- (i) Write down the gain in kinetic energy of the system in terms of  $v$ . [1]
- (ii) Find, in terms of  $x$ ,
- (a) the loss of gravitational potential energy of the system, [2]
- (b) the work done against the frictional force. [3]
- (iii) Show that  $21v^2 = 220x$ . [2]



The diagram shows a vertical cross-section  $ABC$  of a surface. The part of the surface containing  $AB$  is smooth and  $A$  is  $2.5\text{ m}$  above the level of  $B$ . The part of the surface containing  $BC$  is rough and is at  $45^\circ$  to the horizontal. The distance  $BC$  is  $4\text{ m}$  (see diagram). A particle  $P$  of mass  $0.2\text{ kg}$  is released from rest at  $A$  and moves in contact with the curve  $AB$  and then with the straight line  $BC$ . The coefficient of friction between  $P$  and the part of the surface containing  $BC$  is  $0.4$ . Find the speed with which  $P$  reaches  $C$ . [6]



The diagram shows a wire  $ABCD$  consisting of a straight part  $AB$  of length 5 m and a part  $BCD$  in the shape of a semicircle of radius 6 m and centre  $O$ . The diameter  $BD$  of the semicircle is horizontal and  $AB$  is vertical. A small ring is threaded onto the wire and slides along the wire. The ring starts from rest at  $A$ . The part  $AB$  of the wire is rough, and the ring accelerates at a constant rate of  $2.5 \text{ m s}^{-2}$  between  $A$  and  $B$ .

- (i) Show that the speed of the ring as it reaches  $B$  is  $5 \text{ m s}^{-1}$ . [1]

The part  $BCD$  of the wire is smooth. The mass of the ring is 0.2 kg.

- (ii) (a) Find the speed of the ring at  $C$ , where angle  $BOC = 30^\circ$ . [4]

- (b) Find the greatest speed of the ring. [2]